09:32:42 OCA PAD INITIATION - PROJECT HEADER INFORMATION 07/14/88

Active

Project #: E-21-688
Center # : R6530-0A0

Cost share #: E-21-386
Center shr #: F6530-0A0

Rev #: 0
OCA file #:
Work type : RES
Document : GRANT
Contract entity: GTRC

Contract#: ECS-8715364
Prime #:

Mod #:

Subprojects ? : N
Main project #:

Project unit: EE Unit code: 02.010.118
Project director(s):
MELIOPOULOS A P EE

Sponsor/division names: NATL SCIENCE FOUNDATION / GENERAL
Sponsor/division codes: 107 / 000

Award period: 880701 to 901231 (performance) 910331 (reports)

Sponsor amount	New this change	Total to date
Contract value	34,744.00	34,744.00
Funded	34,744.00	34,744.00
Cost sharing amount		1,737.00

Does subcontracting plan apply ?: N

Title: COMPOSITE POWER SYSTEM SIMULATION METHOD

PROJECT ADMINISTRATION DATA

OCA contact: Steven K. Watt 894-4820

Sponsor technical contact

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Security class (U,C,S,TS) : U
Defense priority rating : N/A
Equipment title vests with: Sponsor
NONE PROPOSED

ONR resident rep. is ACO (Y/N): N
NSF supplemental sheet
GIT X

Administrative comments -
INITIATION

5R 8182

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 08/02/91

Project No. E-21-688 — Center No. R6530-0A0

Project Director MELIOPOULOS A P — School/Lab ELEC ENGR

Sponsor NATL SCIENCE FOUNDATION/GENERAL

Contract/Grant No. ECS-8715364 — Contract Entity GTRC

Prime Contract No. ____________

Title COMPOSITE POWER SYSTEM SIMULATION METHOD

Effective Completion Date 910630 (Performance) 910930 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	N	
Final Report of Inventions and/or Subcontracts	Y	910709
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

CommentsBILLING VIA LINE-OF-CREDIT; 98A SATISFIES REQUIREMENT FOR PATENT REPORTING.

Subproject Under Main Project No. ____________

Continues Project No. ____________

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

~~NOTE: Final Patent Questionnaire sent to PDPI.~~ apr

E-21-6??

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GEORGIA 30332

PHONE (404) 894-

May 18, 1989

Dr. Anjan Bose
Program Director
Engineering Systems - Power Systems
National Science Foundation
Washington, DC 20550

RE: Grant ECS-8715364

Dear Dr. Bose:

Please consider this letter as an application for continuation of the above referenced grant. In support of this application, enclosed please find:

1. Summary of Project Accomplishment to this date.

2. Reprints of three papers (published or scheduled to be published).

3. Summary of expended or committed funds.

4. Proposed budget for next year.

An interim report is under preparation. This report will be submitted July 1989.

Thank you for your continuing support of our research.

Sincerely,

A. P. Sakis Meliopoulos
Professor

APM/pk
Enclosures
cc: Roger Webb, Acting Director

SUMMARY

A summary of the first year research effort, in collaboration with Professor G. Cokkinides of the University of South Carolina, is as follows. Three research problems were addressed:

1. Monte Carlo simulation of power system operation.
2. Basic algorithm for composite power system simulation.
3. Contingency ranking/voltage security.

A Monte Carlo simulation method has been developed to provide a benchmark for testing the proposed basic algorithms for composite power system simulation.

Basic algorithms for composite power system simulation have been developed and validated with Monte Carlo methods. These algorithms directly compute probability distribution functions of power system output variables such as circuit flows, bus voltages, transmission losses, etc. Major operating practices such as economic dispatch and nonlinearities, resulting from the power flow equations, are rigorously treated. The method is extremely efficient as compared to an enumerative or Monte Carlo approach. The significance of the method is twofold: (1) direct computation of the probability distribution functions of output variables and (2) efficiency. The method represents a new approach to the probabilistic power flow problem. Detailed description of the method is provided in the attached technical paper entitled, "A New Probabilistic Power Flow Analysis Method," to be presented at the 1989 IEEE-PES Summer Meeting and to be published in the IEEE Transactions on Power Apparatus and Systems.

Contingency ranking methods and voltage security issues have been investigated in the context of reliability analysis. Significant results have been obtained and described in the attached two technical papers: (1) "Corrective Control for Voltage Security," Proceedings: Bulk Power System Voltage Phenomena-Voltage Stability and Security, EPRI Report EL-6183, pp. 8-29 through 8-48, and (2) "A New Contingency Ranking Method," Proceedings of the Southeastcon '89, Columbia, South Carolina, pp. 837-841, April 1989. The significance of these results are summarized as follows: First a contingency ranking method has been developed which eliminates misrankings due to discontinuities arising from reactive power limits and transformer taps and minimizes misrankings due to the nonlinearities of power flow equations. Second, severe post-contingency solutions are corrected by optimal control of reactive power. Together, contingency ranking and corrective control provide the basis for computing voltage security indices.

Future research efforts will concentrate on the following: (1) utilization of the composite power system simulation method for reliability analysis and (2) development of theory for linking corrective control algorithms to the composite power system simulation method. This effort will naturally lead to probabilistic optimization methods.

Grant ECS-8715364

Summary of Expended or Committee Funds

Period: July 1, 1988 through June 30, 1989

Senior Personnel	
Dr. A. P. Sakis Meliopoulos	$ 5,801
Graduate Students	
Ms. Carol Cheng	$ 4,320
Mr. Xing Yong Chao	6,180
Travel	$ 1,222
Materials and Supplies	$ 165
Fringe Benefits	$ 1,480
TOTAL DIRECT COSTS	$19,168
Indirect Costs (Overhead)	$11,500
GRAND TOTAL	$30,668

Final Report

Grant ECS - 8715364
and
Grant ECS - 8715038

Composite Power System Simulation Method

Prepared by

A. P. Meliopoulos, Co-principal Investigator
G. J. Cokkinides, Co-principal Investigator
X. Y. Chao, Graduate Research Assistant
C. Cheng, Graduate Research Assistant
F. Xia, Graduate Research Assistant
F. Schroer, Undergraduate Research Assistant

GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical Engineering
Atlanta, Georgia 30332

and

UNIVERSITY OF SOUTH CAROLINA
Department of Electrical Engineering
Columbia, South Carolina 29208

Prepared for

NATIONAL SCIENCE FOUNDATION
Engineering Division / ECS
Washington, DC 20550

June 1991

Final Report

Grant ECS - 8715364
and
Grant ECS - 8715038

Composite Power System Simulation Method

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Washington, DC 20550

June 1991

Table of Contents

Composite Power System Simulation Method

1. Executive Summary

New methodologies for simulation of the composite power system and related models have been introduced and successfully tested and validated on actual power systems.

Simulation of the operation of a composite power system is of great importance for expansion and operation planning studies. Recent trends, resulted from the 1978 PURPA legislation, such as projected proliferation of customer owned generation, alternate electric energy sources, possible deregulation of the power industry, and others have accentuated the necessity of sophisticated simulation methods.

Simulation methods for thermal generation systems have been well developed and production grade programs are available. However, mathematically rigorous methods for the simulation of the integrated generation and transmission system do not exist. As a result, there is a gap in the existing simulation methods and a lack of rigorous solutions to problems dependent on the operation of the composite power system. The composite power system simulation problem has been addressed in this project.

During the course of the project, a new methodology was introduced for the simulation of the composite power system. The methodology is based on stochastic models of the following components:

a) Future Electric load
b) Power System Equipment Availability
c) Customer Owned Generation

d) Power Wheeling Schedules
e) Other variables affecting power system operation

A decomposition method has been investigated by which the simulation procedure is decomposed into two components:

a) The simulation of the generation system
b) The simulation of the transmission system

The two simulation components are connected through a probabilistic optimization model which takes into account effects of transmission constraints, interchange transactions, etc.

The simulation of the generation system is based on an extension of the well known probabilistic simulation method, which incorporates most operational practices and constraints such as economic dispatch, forced outages, etc. The result of the simulation is a probabilistic model of bus power injections. The bus power injection model is utilized in the simulation of the transmission system. The transmission system simulation is based on a new generalized probabilistic power flow model. This model accurately represents network nonlinearities, circuit random outages, etc. The overall simulation approach is illustrated in Figure 1. The developed simulation method was applied to two specific problems: (a) reliability assessment and (b) transmission loss evaluation. Many other related topics were addressed which resulted in a model with expanded capabilities.

The method has been validated by means of Monte Carlo simulation. The method was applied to the system of the local utility which is a rather large system (Georgia Power Company) thus verifying the applicability of the method to large scale systems.

The achievements of this project has been archived in the following technical papers:

1. A. P. Sakis Meliopoulos, George J. Cokkinides, and Xing Yong Chao, "A New Probabilistic Power Flow Analysis Method" IEEE Transactions on Power Systems, Vol. PWRS-5, No. 1, pp 182-190, February 1990

2. A. P. Meliopoulos, X. Chao, George J. Cokkinides, R. Monsalvatge, "Transmission Loss Evaluation on Probabilistic Power Flow" IEEE Transactions on Power Systems, Vol. PWRS-6, No. 1, pp 364-371, February 1991

3. A. P. Sakis Meliopoulos and Carol Cheng, "Corrective Control For Voltage Security" The NSF workshop on Bulk Power System Voltage Phenomena, Potosi, Missouri, September 18-24, 1988

4. A. P. Sakis Meliopoulos and Carol Cheng, "A New Contingency Ranking Method," Proceeding 1989 Southeastcon, Vol.2, pp.837-842, Columbia, South Carolina, April, 1989

5. A. P. Sakis Meliopoulos and C. Cheng, "A Hybrid Contingency Ranking Method," Proceeding of the 10th Power System Computation Conference, Graz, Austria, Aug. 1990.

6. C. Cheng and A. P. Sakis Meliopoulos, "Performance Evaluation of a Subnetwork Solution for Contingency Selection," The Proceeding of the 22nd Annual North American Power Symposium, pp. 348-362, Auburn, Alabama, October 15-16, 1990.

7. A. P. Sakis Meliopoulos and George J. Cokkinides, "Modeling and Optimization Issues In Expansion Plan Evaluation Methods," Proceedings of the NSF Workshop on Research Needs in Power System Operations and Planning, Atlanta, Georgia, September 5-8, 1991.

8. A. P. Sakis Meliopoulos, Feng Xia, and Xing Yong Chao, 'Probabilistic Analysis and Control of a less Regulated Power System', Presented at the NSF Workshop on Impact of a Less Regulated Utility Environment on Power System Control and Security, University of Wisconsin, Madison, April 19-20, 1991.

9. A. P. Sakis Meliopoulos and Xing Yong Chao, "Non-Divergent and Optimal Power Flows: A Unified Approach," Submmited to 1992 IEEE-PES Winter Meeting

10. A. P. Sakis Meliopoulos and Feng Xia, "An Analytic Method for Composite Power System Simulation" Submitted to The 29th North American Power Symposium, Carbondale, Illinois, October 7-8, 1991

Student involvement and contributions were substantial. A brief summary of student involvement and contributions follow:

Ms Carol Cheng has concentrated on the problem of identifying contingencies which impact system reliability and the associated problem of voltage collapse. She has completed her Thesis entitled "A Hybrid Approach to Power System Voltage Security Assessment". She plans to defend her Thesis in the Summer of 1991.

Mr Xing Yong Chao has focused on the transmission simulation method and the associated optimization problems. He is presently writing his PhD thesis entitled "Non-Divergent and Optimal Power Flow - A Unified Approach". He plans to defend his Thesis in the Summer of 1991.

Mr Feng Xia has focused on the probabilistic power flow problem. Presently he is on his second year of the PhD program. He plans to continue work on the probabilistic power flow.

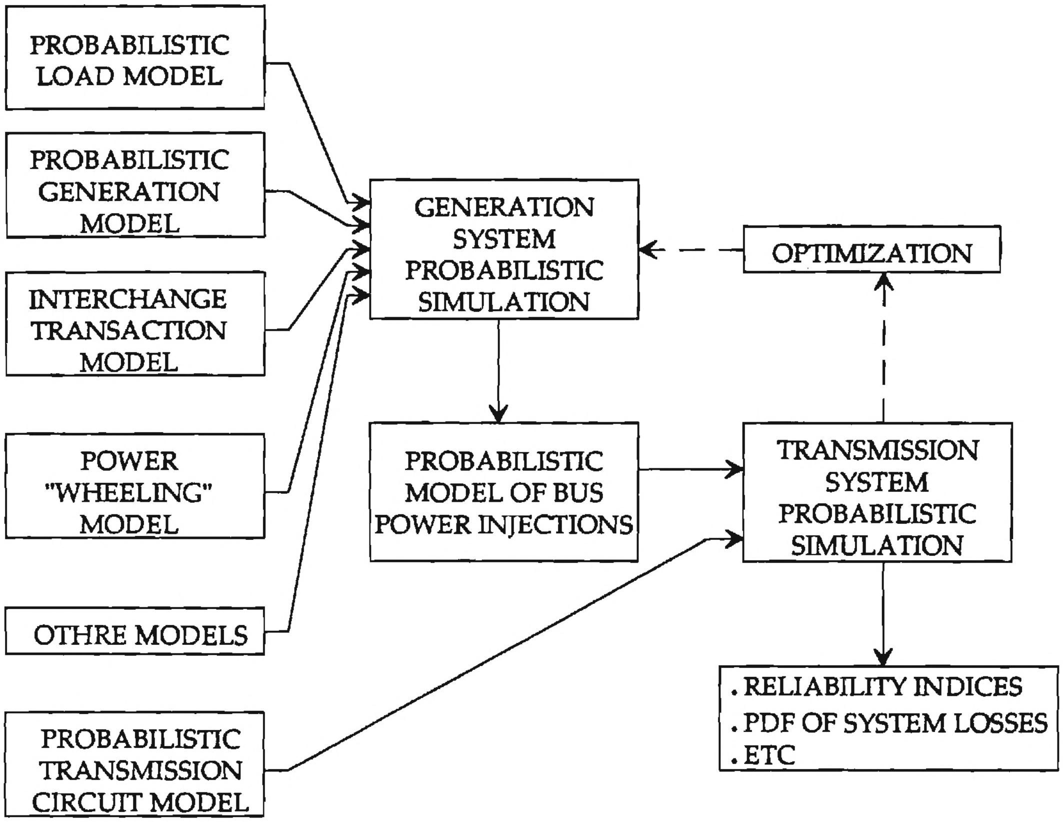

Figure 1. Schematic Representation of the Composite Power System Simulation

Mr Frank Schroer, an undergraduate student, worked on the research project assisting in various ways. Mr Schroer graduated in December 1990.

2. Brief Description of Research Accomplishments

A number of technical papers were written and published during the course of the project. The papers describe the research accomplishments and they are attached in this report as Appendices A through J.

A brief discussion of the research accomplishments described in each of these papers follows.

The paper entitled "A New Probabilistic Power Flow Analysis Method" proposes a new simulation method of the composite power system for the purpose of evaluating the probability distribution function of circuit flows and bus voltage magnitudes. The method consists of two steps. First, the electric load and generating unit is substituted with random variables, Y, which represent power injections at system buses. The statistics of the power injections are computed by direct application of probability theory and by simulating major operating practices of the power system such as economic dispatch. Subsequently, a probabilistic power flow provides the probability distribution function of circuit flow and bus voltage magnitudes from the probabilistic model of the injection variables Y. This step consists of expressing circuit flow and bus voltage magnitudes as a linear combination of the random variables Y and computation of their statistics from the statistics of Y. The effects of nonlinearities due to power flow equations are accounted for by partitioning the electric load into a small number of segments and using linearized models around the mean value of each load segment. Validation of the method is performed via Monte Carlo simulation. Typical results illustrate that the proposed method matches very well results obtained with Monte Carlo simulations. Potential applications of the proposed method are: (1) composite power system reliability analysis and (2) transmission loss evaluation.

The application of this method for transmission loss evaluation is described in the technical paper entitle "Transmission Loss Evaluation Based on Probabilistic Power Flow".

Another accomplishment of the project involves a new contingency ranking method. This method can accurately classify contingencies according to a specified performance index. An advantage of the new method is that it effectively addresses the problem of misranking due to system nonlinearities and control variable limits and discontinuities. This method greatly improves the efficiency of stochastic simulation method by limiting the examined contingencies to a small set containing the cases that have the strongest influence on the parameters of interest. Papers describing this method in detail are included in Appendices C, D, E and F.

The paper entitled "Corrective Control For Voltage Security" proposed an efficient contingency ranking method to detect voltage problem coupled with a corrective control strategy to maximize voltage security. Voltage security encompasses present operating conditions and probable disturbances. For voltage security assessment, a new method is proposed which explicitly models the effects of voltage regulators. For corrective controls, a method is proposed which may alleviate or minimize voltage insecurity. Specifically, a problem formulation is proposed which leads to an optimal control problem. This problem is a large nonlinear optimization problem. A solution method is proposed based on a successive linear programming approach. The major innovations of the methodology are: (1) a model reduction method which is based on coherency analysis of problem constraints, and (2) a methodology to linearize the problem and to define the region of validity of the linearized model. This methodology allows the solution of this nonlinear model with successive linear programming. Results on the 24 bus IEEE reliability test system are provided. Solution times of the proposed method are comparable to those from a power flow.

The paper entitled "A New Contingency Ranking Method" proposes a new contingency ranking method for detecting voltage problems. The method is based on the AC network model and a voltage sensitive performance index (PI). An efficient algorithm is proposed for the computation of the change of the performance index with respect to each contingency. The proposed method has the following innovations: (1) voltage regulators are explicitly modeled, (2) any form of the performance index can be accommodated, (3) discontinuities in the power system model arising from generating bus reactive power limits and regulator tap limits are effectively addressed, and (4) nonlinearities of the power flow equations with respected to contingency parameters are accounted for by the introduction of the stiffness index. The concept of bus stiffness provides a priori knowledge of the accuracy of the ranking method. The methodology has been applied to the IEEE reliability test system. The computational requirements of the proposed method are comparable to that of one iteration of the Newton-Raphson power flow. The proposed method is suitable for on-line voltage security assessment.

The paper entitled "A Hybrid Contingency Ranking Method" proposes a hybrid ranking method for detecting voltage problems. The method is based on the AC network model of a power system. It employs an algorithm to classify contingencies into two groups. The first group comprises contingencies which can be effectively ranked with a performance index method, while the second group comprises contingences which cause major nonlinearities. These contingencies can be effectively ranked with a subnetwork solution method. A two step method is proposed for identification of contingences which must be ranked with the subnetwork solution method. First, a performance index method is employed to predict which contingences will cause discontinuities in the unit reactive power limits or transformer tap limits. Second, the concept of contingency stiffness is introduced to predict the severity of nonlinearities of the reactive power equations. The remaining contingencies are ranked with a PI based method. The proposed method accounts for the effects of (1) voltage regulators, (2) discontinuities arising from generation reactive power limits or transformer tap limits, and (3) voltage dependent load. Test results of three power systems are provided. The computational requirement of the proposed method are comparable to that of three iterations of the Newton-Raphson power flow.

The paper entitled "Performance Evaluation of a Subnetwork Solution for Contingency Selection" examines the computational efficiency of the subnetwork solution for the contingency selection. A statistical approach is used to evaluate the performance of the subnetwork solution method using two different size system: (a) a northeastern utility's 308 bus system and (b) Georgia Power Company's 1304 bus system. For the purpose to evaluate the performance of the subnetwork solution, the sparsity-oriented subnetwork solution with the fast forward and fast back (FF/FB) substitution is referred to as the subnetwork solution, while the conventional power flow solution with the full forward and back substitution is referred to as the direct solution. Performance evaluation of the subnetwork solution yielded the following observations: (1) When the mismatch vector is sparse and the solution vector is also fairly sparse, the subnetwork solution is substantially superior to the direct solution. (2) The computational advantage of the subnetwork solution is dependent upon the size of the local network. (3) The advantage of the subnetwork solution is more significant for larger systems than smaller systems, but depending on the system structure. (4) The computational efficient of the subnetwork solution with FF/Full back is between the one with FF/BB and the direct solution. The results of the performance evaluation show that the subnetwork solution method provides a promising approach for contingency selection.

The paper entitled "Modeling and Optimization Issues in Expansion Plan Evaluation Methods" discusses modeling and optimization issues as dictated by recent trends and developments in the electrical power industry. First, an enhanced model of the electric load is proposed which is based on the multiple input / multiple output ARIMA model and addresses the following concerns: (1) power wheeling schedules, (2) customer owned generation and (3) electric load modulation due to specific rate structures. Second, a new power flow/optimization formulation is proposed which addresses the following concerns: (1) combines the remedial actions with the power flow solution, (2) eliminates the necessity of adjustments during the power flow solutions, (3) guarantees that severe power mismatches will not result in nonconvergent power flows. Third, composite power system simulation methods such as the enumerative approach, Monte-Carlo simulation and probabilistic simulation method have been reviewed. A new probabilistic simulation method is proposed which provides the probability distribution function of specific system attributes by incorporating major operation practices. Finally, a sensitivity analysis embedded in a composite power simulation method provides measures of controllable device effectiveness on any pertinent power system attributes such as operating cost, security, and transmission losses.

The paper entitled "Probabilistic Analysis and Control of a Less Regulated Power System" proposes new tools for power system analysis and control which are suitable to study the effects of increased deregulation and competition. A comprehensive model of the non-utility system which consists of electric loads, independent power producers, wheeling customer, etc has been proposed. Three probabilistic simulation methods, Monte Carlo simulation, analytic approach (probabilistic power flow) and enumerative approach are presented and discussed. Finally, the paper proposes a static simulation method of power system operation which combines the traditional power flow remedial actions, and optimal power flow in one unified approach and solution of the non-divergent and optimal power flow.

The paper entitled "Nondivergent and Optimal Power Flow: A Unified Approach" presents a new formulation of the power flow problem which combines the traditional power flow, remedial actions, and optimal power flow in one unified approach. The formulation and solution are based on mathematical programming techniques that incorporate the process of economic dispatch, and observe the operating constraints. The method introduces fictitious generators at each bus which represent the power mismatches. The output of the fictitious generators is reduced slowly while the system state is steered along in a trajectory which maintains feasibility and optimality. The process guarantees convergence, if a solution exists, and optimality with respect to a specified objective function. The proposed method has been tested with the 24 bus IEEE reliability test system and 1304 bus Georgia Power Company bulk power system. The efficiency of the method is competitive with the usual power flow algorithms.

The paper entitled "An Analytic Method for Composite Power System Simulation" proposed an analytic simulation method of the composite power system analysis which takes into consideration the uncertainty of the electric loads, availability of generating units and transmission lines, nonlinearities in the power flow equations, as well as the major operating practices such as economic dispatch. The method is based on the following procedure. First, given the probabilistic electric model, the probability distribution function of power injections at generation buses is computed as a function of a small number of independent random variables by simulating generating unit forced outages and economic dispatch practices. Next, circuit flows, bus voltage magnitudes, etc are expressed as linear combinations of power injections at generation buses. This relation allows the computation of the distribution functions of circuit flows, bus voltage, transmission losses, etc. The method has been validated by comparing it to the exact results for a three bus system calculated by complete enumeration.

3. Future Research Extensions

The investigations of this project demonstrated that the proposed simulation method is a very effective tool for the simulation of the composite power system. The method is capable of simulating the operation of the system by taking into consideration random and planned outages of generating units, random variations of electric loads, and economic operational practices of power systems. The simulation applies to a specific state of the transmission system. Thus the method should be applied to each credible transmission outage and the results be processed to compute reliability indices. The number of credible transmission outages is normally very large. An improvement in selecting a set of credible transmission outages can be effected by utilizing proper contingency ranking methods. A new hybrid contingency ranking method has been developed and successfully tested within this research project. Still, utilization of the contingency ranking method will yield a rather large number of credible transmission outages.

The results of this research project suggest that additional work is necessary to utilize the composite power system simulation method for reliability analysis. Specifically, it is necessary to develop methods for the systematic identification of transmission outages which contribute to system unreliability. Such an approach will consist of determining the transmission outages which cause violation of specific failure criteria. This amounts to identifying minimal cut states. A minimal cut state is defined in this case as a state with the following property: Given a failure criterion, a state is minimal if and only if failure of a single transmission element, any element, will lead to violation of the failure criterion. It should be apparent that identification of all minimal cut states will enable the computation of reliability indices as defined in the literature. The number of minimal cut states may be large. But in any case, the number of minimal cut states is smaller than the number of possible transmission outages. The developed methodologies under this project are well suited for achieving this goal. Specifically, the nondivergent optimal power flow is a tool designed specifically to identify transmission outages which will lead to violation of a specified criterion. Thus the nondivergent optimal power flow serves the purpose of identifying the minimal cut states. It should be noted that the traditional power flow or an optimal power flow is not suitable for this purpose, since it is required that the method determines whether the system is adequate to serve the load. A search algorithm can be devised for this purpose. The essentials of the search algorithm are illustrated in Figure 2. Once the minimal cut states have been identified, the composite power system simulation method can be used to determine the expected indices of performance of the system for reliability index computations. The process is illustrated in Figure 3. The figure illustrates the concept for the computation of both expectation indices and frequency and duration indices. Each contingency is evaluated with the composite power system simulation method. In all of these methods, the Monte Carlo simulation method should play an important role as the method to be used for validation of the analytic methods.

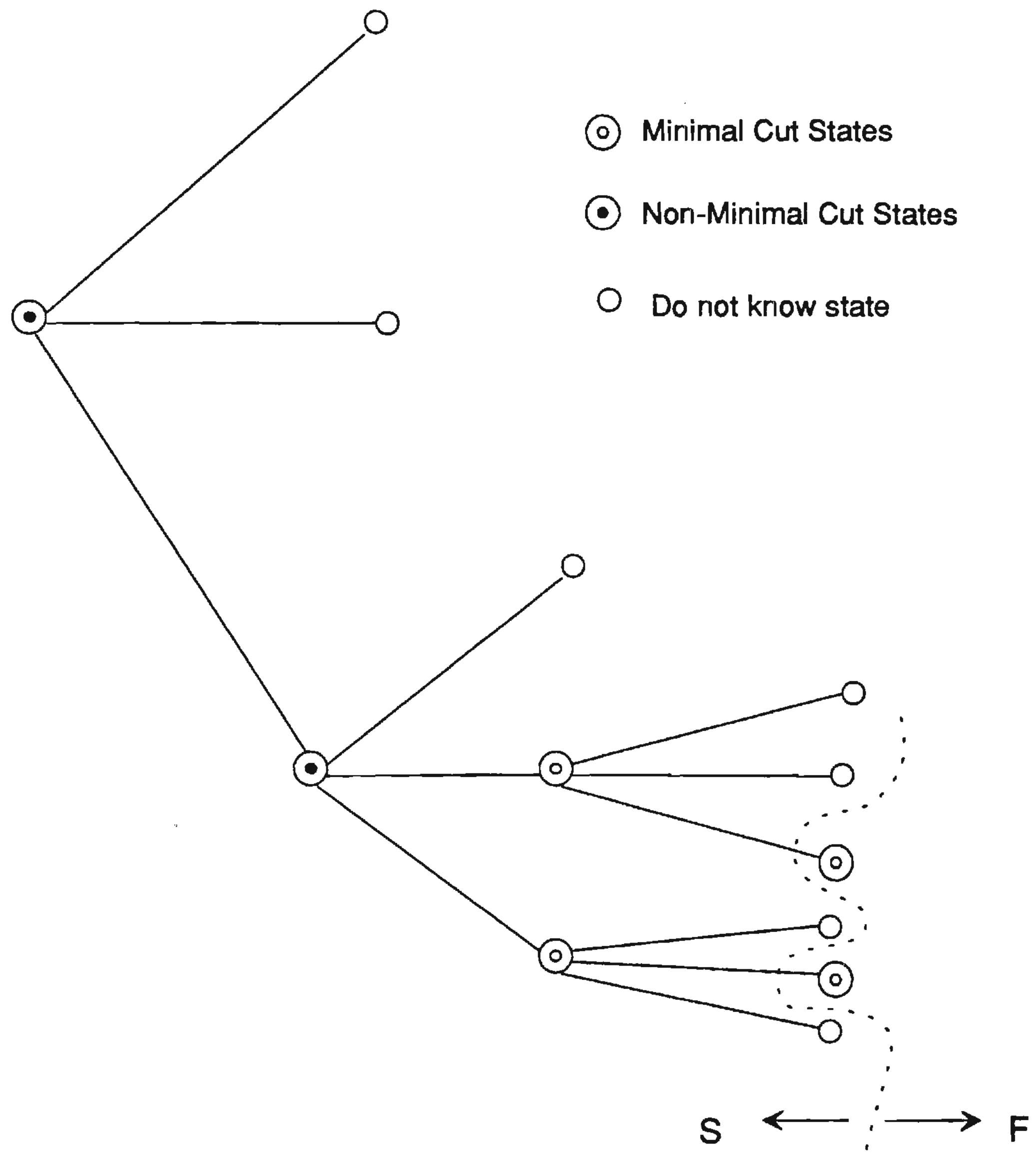

Figure 2. Illustration of Search Algorithm
for Minimal Cut States

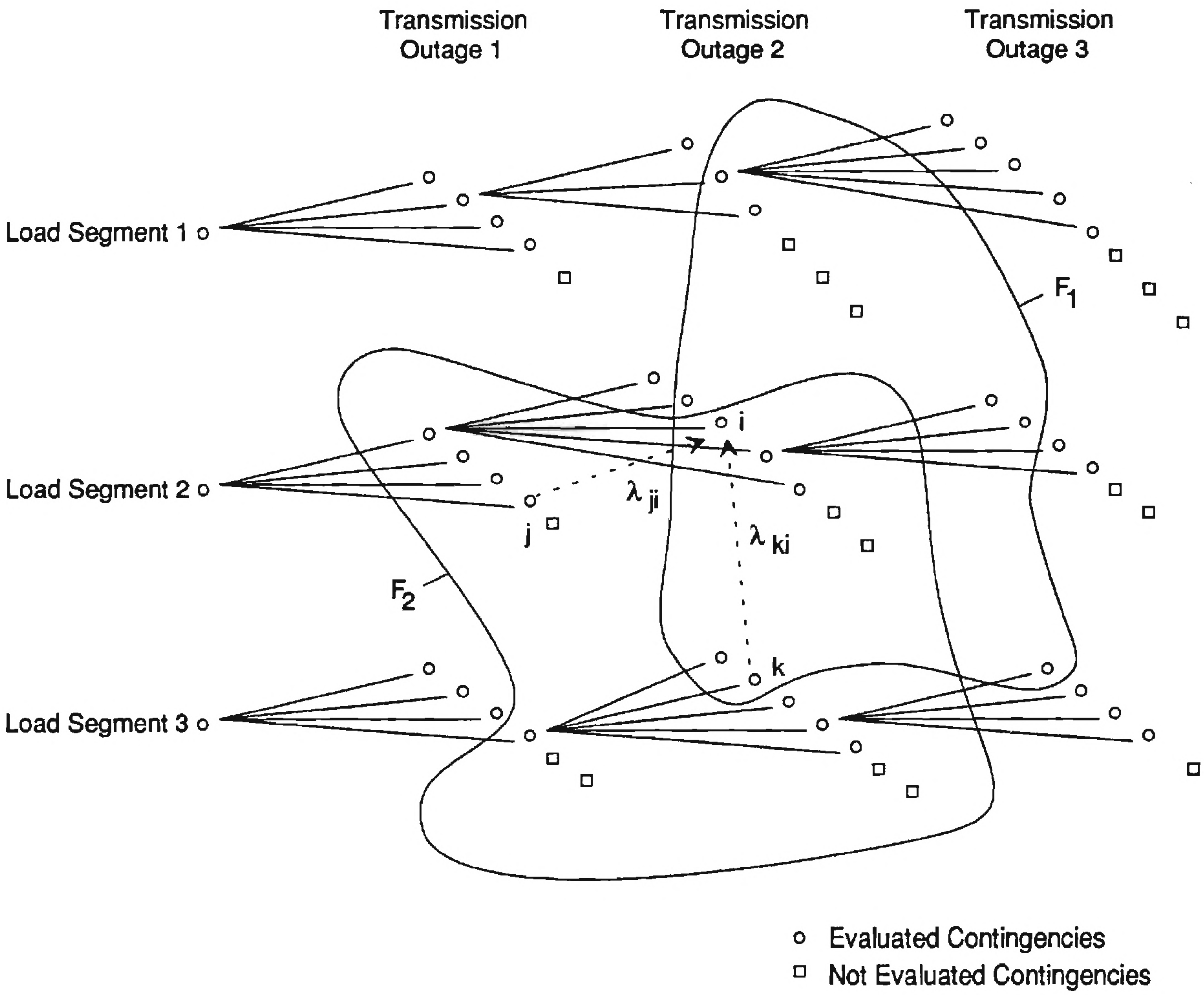

Figure 3 Enumeration of Transmission Outages and Load Segments for Composite Power System Reliability Assessment

APPENDIX A : A New Probabilistic Power Flow Analysis Method

A. P. Sakis Meliopoulos, George J. Cokkinides, and Xing Yong Chao, 'A New Probabilistic Power Flow Analysis Method' IEEE Transactions on Power Systems, Vol. PWRS-5, No. 1, pp 182-190, February 1990.

A NEW PROBABILISTIC POWER FLOW ANALYSIS METHOD

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Abstract

A simulation method of the composite power system is proposed for the purpose of evaluating the probability distribution function of circuit flows and bus voltage magnitudes. The method consists of two steps. First, given the probabilistic electric load model, the probability distribution function of the total generation of generation buses is computed. Second, circuit flows and bus voltage magnitudes are expressed as linear combinations of power injections at generation buses. This relationship allows the computation of the distribution functions of circuit flows and bus voltage magnitudes. The method incorporates major operating practices such as economic dispatch and nonlinearities resulting from the power flow equations. Validation of the method is performed via Monte Carlo simulation. Typical results are presented which illustrate that the proposed method matches very well results obtained with Monte Carlo simulations. Potential applications of the proposed method are: (1) composite power system reliability analysis and (2) transmission loss evaluation.

Key Words

Power Flow
Economic Dispatch
Stochastic Load Model
Probability Distribution Function (PDF)
PDF of Circuit Flow
PDF of Bus Voltage
Monte Carlo Simulation

Introduction

Traditional power flow analysis treats the electric load and the generating units of the system as deterministically known quantities. This is only true for a limited number of situations, for example, in a real time environment where the electric load and generation can be directly measured. In any other power flow application, however, there is uncertainty associated with the availability of generating units and the electric load. In many applications, such as reliability analysis of the composite (generation and transmission) power system and transmission loss evaluation, use of the traditional power flow formulation leads to an extremely large number of power flow cases for the purpose of capturing all the variances of the electric load and generation dispatch schedules. In these cases, it is appropriate to use methods which directly treat the uncertainty. Methods of power flc analysis which recognize the uncertainty of th generation and electric load are referred to s probabilistic power flows.

The first notion of probabilistic power flc appeared in the early 1970s. Borkowska, Allen et al [14,15] have proposed a simplified probabilistic loa flow. Two assumptions were introduced: (1) th electric power system is represented with a DC networ model (thus, the reactive power flow is neglected) and (2) the real part of the bus electric loads ar independent random variables. With these assumptions a conventional deterministic power flow is solve first, assuming net nodal loads equal to their mea values. This solution determines the operating poin about which the load flow equations are subsequentl linearized. Within this model, the generation dispatc procedure is modeled with an arbitrary function whic allocates the variation of the total electric load t the specific generation buses. Since the variables o the nodal electric load are assumed independent, th probability density functions of the circuit flows ca be computed with a series of convolutions. Later, thi basic method has been extended to the AC network mode [18].

The assumption of independence of the noda electric loads is unrealistic. Da Silva et al. pro posed a linear dependence model of electric loads [19] Using a linearized power flow model, they proposed method which combines Monte Carlo simulation an convolutions. Dopazo et al. [16] proposed a metho which models the correlation between the load at an two buses. Their proposed method assumes that circui flows and bus voltage magnitudes are Gaussian distrib uted and, thus, only the variance must be computed Monte Carlo simulations indicate that it is unrealisti to assume Gaussian distributions of circuit flows an bus voltages. For this reason, Sauer and Heydt [20 have proposed the use of higher moments (third an fourth) for accurate representation of the probabilit distribution functions.

An efficient method for treating the correlatio among bus loads and the generation dispatch procedur has been proposed in [21]. The model assumes Gaussia distribution of bus loads and a linearized economi dispatch model. The circuit flows and bus voltages ar expressed as a linear combination of the bus load only. The linearized equations are utilized to deter mine the moments of probability density function o circuit flows and bus voltages. The inclusion of thi model in a reliability analysis method resulted in mor accurate representation of the electric load at reduce computational requirements [21]. While this approac models the economic redispatch of generating units du to electric load variations, it is based on the linear ized power flow equations and the linearized economi dispatch model. As such, its applicability is limited This paper presents a new approach for this model whic addresses three important aspects: (1) the economi dispatch of generating units, (2) the effects o nonlinearities of the power system model, an (3) the uncertainty associated with the availability o generating units. Validation of the method via Mont Carlo simulation is also presented.

89 SM 714-7 PWRS A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1989 Summer Meeting, Long Beach, California, July 9 - 14, 1989. Manuscript submitted September 1, 1988; made available for printing May 9, 1989.

is a generalization of the load duration curve used in generation system reliability analysis. Observe that for $m = 1$, above model is exactly the load duration curve. Specifically, when $m = 1$, the distribution function, $L_o(\ell)$, is a function of one random variable only, v_i, i.e.

$$\ell = a_o + a_1 v_1$$

In this case the distribution function, $L_o(\ell)$ versus ℓ (or v_i) can be plotted yielding the load duration curve. A typical function, $L_o(\ell)$, in this case is illustrated in Fig. 2.

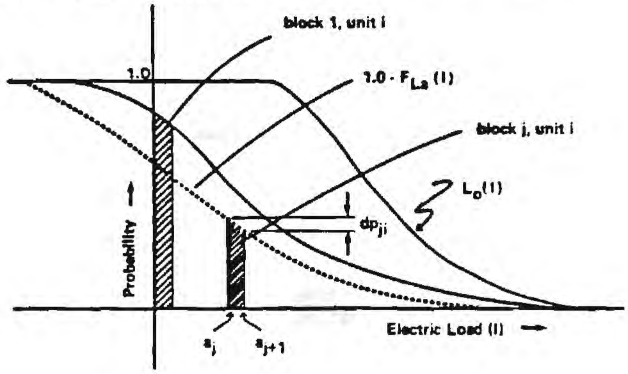

Figure 2. Illustration for the Computation of Unit Probability Distribution Function.

Generation System Model

A generating unit, i, of capacity c_i MW is modeled with a set of capacity states, each state with a specified probability. The probability density function of this model is expressed with

$$f_{X_i}(x_i) = \sum_{k=0}^{m_i} p_{ik}\delta(x_i - d_{ik})$$

where

- m_i is the number of capacity states (including zero and full capacity of the unit)
- d_{ik} are the capacities of the states (note, $d_{io} = 0.0$ and $d_{im_i} = c_i$).

For $m_i = 1$, this model is the well known up and down model. For clarity of presentation, the method is discussed in terms of the up and down model of a unit. Unit outages are independent. In addition to this model, each unit is described with a production cost function versus unit output. This function can be a quadratic function or a piecewise linear function.

Generation System Simulation

The problem of generation system simulation is defined as follows. Given the probabilistic electric load model for the time period under consideration and a list of available generating units, simulate the operation of the system in order to compute the probability distribution functions of the bus power injections and their correlations. The process should account for the effects of economic scheduling functions within the time period considered and the random forced outages of the units.

Given the load and generation models, the following production quantities can be computed with the classical probabilistic method [1]:

(1) Probability of operation of unit i: Pr[Unit i in operation] = Pr[Unit i Output > 0].
(2) Expected value of produced energy from the unit.
(3) Expected value of cost of operation of unit i.

Refinements of this method have been developed over the years. The refinements can be classified into two groups. In the first group, the objective of the refinements is to speed up the computerized procedure of the simulation method. Very fast procedures have been developed based on the cumulant method [6-8, 10-13]. In the second group, the objective is to improve the simulation method of operating practices such as economic dispatch, maintenance, etc. Procedures for simulating incremental loading of units based on economic criteria have been developed [3-5,9]. All these refinements can be incorporated in the proposed probabilistic power flow. For clarity of presentation, we shall use the method described in Ref. [9] to present the probabilistic power flow. This method is briefly described as follows. Consider units of the system operating at levels $x_1, x_2, \ldots, x_n$. If unit k is not in operation, then obviously x_k will equal 0. Since there is a finite probability that an unit can be forced out, the output of unit i, x_i, can be considered to be a random variable with probability of unavailability equal to q_i. We write

$$Pr(X_i = x_i) = 1 - q_i \quad , \quad x_i \neq 0 \qquad (7)$$

$$Pr(X_i = 0) = q_i \qquad (8)$$

where X_i is a random variable representing the generation of unit i. Assume that the electric load equals ℓ. For this condition, the apparent load ℓ will be

$$\ell_a = \ell - x_1 - x_2 - \ldots - x_n \qquad (9)$$

Since $\ell, x_1, \ldots, x_n$ are not deterministically known, the above equation can be replaced with its equivalent equation in terms of the corresponding random variable

$$L_a = L - X_1 - X_2 - \ldots - X_n \qquad (10)$$

where L is a random variable representing the electric load and X_i is a random variable representing the output of unit i. Since the probability distribution functions of the random variables $L, X_1, \ldots, X_n$ are known and since these random variables are independent the probability distribution function of the random variable L_a is computed with a series of convolutions.

If we assume that $\ell_a > 0$ (that is, load exceeds generation), then another unit should be brought in operation or one or more of the operating units should increase their output. Assume that unit i is operating at x_i and that it is selected according to a criterion to respond to any increases in the load. When the criterion is selected to be the incremental production cost of the unit, then the described procedure simulates the economic dispatch practice. In general if $\ell_a > 0$, the output of unit i will increase from x_i to $x_i + \Delta x_i$, where Δx_i is a small increment (1-5 MW). We shall refer to this increment as the block Δx_i. It is noted that if $x_i = 0$, the increment Δx_i may not be small. In this case, unit i will be brought in operation at a level at least equal to minimum allowable operating level. With the described formulation and application of basic probability theory, the expected energy to be produced and cost of operation and required fuel are computed as follows:

Model Description

The proposed model provides probabilistic characterizations of circuit flows and bus voltage magnitudes for a given electric load and generation system model. Specifically, consider a power system as is illustrated in Fig. 1a. The following assumptions are made:

(1) A probabilistic electric load model is given.
(2) The generating unit parameters and forced outage rates are known.
(3) The transmission system is known.

Under these assumptions, it is desired to compute the probability distribution function of circuit flow S_ℓ and bus voltage magnitude V_i for each circuit ℓ and bus i. Major operating practices, such as economic dispatch, must be considered.

The stated objective is achieved with a two step model. In the first step, the electric load and generating system model is used to characterize the power injections, Y, at the system buses as random variables. This is illustrated in Fig. 1b. The random variables, Y, are in general correlated. Subsequently, a probabilistic power flow provides the probability distribution function of S_ℓ, V_i from the probabilistic model of the injections Y.

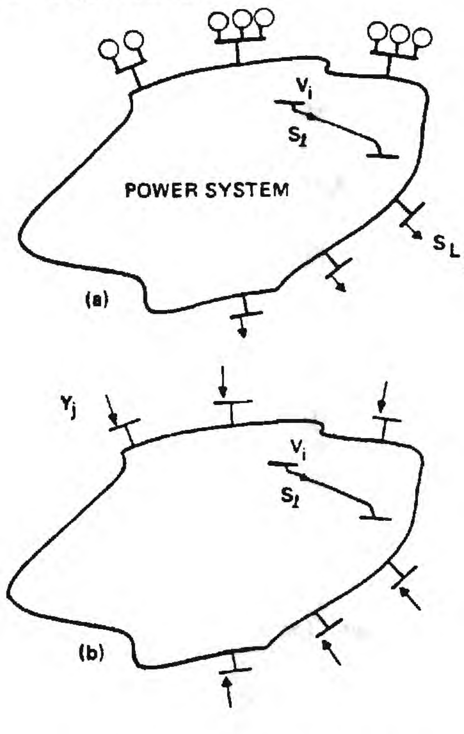

Figure 1. Schematic Representation of an Electric Power System.

The Probabilistic Electric Load Model

The electric load model provides a probabilistic description of system bus loads and allows modeling of conforming or nonconforming bus loads. The assumptions of the probabilistic electric load model are: (1) Bus electric loads are typically strongly correlated. It is, therefore, reasonable to assume that they are generated as a linear combination of a small number of independent stochastic processes. (2) The power factor of the electric load at a specific bus is constant. The equations describing the electric load model for a system with n buses are:

$$\phi_1(B)z(t) = \phi_2(B)n(t) \tag{1}$$

$$z(t) = \phi_3(B)v(t) \tag{2}$$

$$P(t) = P_o + Av(t) \tag{3}$$

$$S_i(t) = P_i(t) + j\alpha_i P_i(t) \tag{4}$$

where

$n(t)$	is an m-vector of independent white noise processes
$z(t)$	is an m-vector of stationary stochastic processes
$v(t)$	is an m-vector of nonstationary stochastic processes
$P(t)$	is an n-vector of bus electric loads (real power)
ϕ_1,ϕ_2,ϕ_3	are vector functions of arbitrary polynomials
B	is the backward operator
P_o	is a constant n x 1 vector
A	is an n x m matrix
$S_i(t)$	is the complex electric load at bus i
α_i	is a constant for bus i; it is dependent upon the power factor of the load at bus i.

The model described with Eqs. (1) and (2) (ARIMA model) has been extensively used to represent the electric load. For example, see References [24,25]. It is well known that it is capable of representing the periodicities as well as the nonstationary property of the electric load. The innovation introduced here is the linear model A which translates the low order nonstationary stochastic process vector v(t) into the vector P(t) of the bus electric loads. The number of independent processes v(t), m, is in general low. For a system with conforming bus loads, m is equal to one. The total electric load is the summation of all bus loads:

$$\ell(t) = \sum_{i=1}^{n} P_i(t) = a_o + \sum_{i=1}^{m} a_i v_i(t) \tag{5}$$

Above equation provides the total electric load, $\ell(t)$, at time t as a function of the stochastic process array v(t). The model has been structured in such a way that the stochastic processes v(t) are normalized, i.e. they assume values in the interval (0,1). For probabilistic power flow applications, it is necessary to characterize the total electric load at a specified future time or at a specified future interval (for example, one week, one month, one year interval). For a specified time interval, T, which shall be referred to as the simulation time, the stochastic processes v(t) and $\ell(t)$ are replaced with random variables V and L. Then Eq. (5) becomes

$$L = a_o + \sum_{j=1}^{m} a_i V_i \tag{6}$$

The statistics of the random variables V can be obtained from the ARIMA model (1) and (2). From the known statistics of V, the probability distribution function, $F_L(\ell)$ of L, are computed. The complementary distribution function, $L_o(\ell)$, of the total load is defined with

$$L_o(\ell) = 1.0 - F_L(\ell) = \Pr[L > \ell]$$

The complementary distribution function, $L_o(\ell)$, depends on m independent random variables v_i, i = 1,2,...,m. It should be pointed out that this model

Step 1: Compute the probability distribution function of random variable

$$L' = L_a + X_i$$

Let it be $F_{L'}(z)$. If $x_i = 0$, skip this step and assume $F_{L'}(z) = F_L(z)$. (Note that this step requires a deconvolution.)

Step 2: Compute

$$E(\Delta x_i) = (1-q_i)T \int_{z=x_i}^{x_i+\Delta x_i} \left(1-F_{L'}(z)\right)dz \qquad (11)$$

$$C(\Delta x_i) = (1-q_i)Tf_i(0)\left(1-F_L(0)\right)\delta(x_i)$$

$$+ (1-q_i)T \int_{z=x_i}^{x_i+\Delta x_i} \frac{df(z)}{dz}\left(1-F_L(z)\right)dz \qquad (12)$$

where

$\delta(x_i) = 1$ if $x_i = 0$, $\delta(x_i) = 0$ if $x_i \neq 0$

$f(z)$ = production cost function of unit i

T = simulation time period (in hours)

$E(\Delta x_i)$ = expected energy to be produced from block Δx_i

$C(\Delta x_i)$ = expected cost of operation of block Δx_i.

Probability Distribution Function of Unit Output

This function for unit i is defined with:

$$\Pr[X_i < \alpha] = F_{G_i}(\alpha) \qquad (13)$$

It is computed by considering the contributions from individual blocks of the unit i. As an example, consider the loading of block j of unit i. Assume that this block is loaded in such an order that it "sees" the equivalent load ℓ_a. The complementary probability distribution function of the equivalent load ℓ_a is illustrated in Fig. 2 (dotted curve). The probability distribution function of X_i is computed as follows. Consider the following identity:

$$\Pr[X_i < \alpha] = \Pr[X_i < \alpha | E_i]\Pr[E_i]$$

$$+ \Pr[X_i < \alpha | \overline{E}_i]\Pr[\overline{E}_i]$$

where E_i is the event that unit i is available. $\overline{E}_i$ is the complimentary event of E_i. Application of above equation for block j of unit i yields

$$\Pr[X_i < \alpha] = q_i + p_i F_{L_a}(\alpha) \quad , \quad \alpha_j < \alpha < \alpha_{j+1} \qquad (14)$$

where α_j, α_{j+1} are the limits of block j, unit i, and $p_i = \Pr[E_i]$. Equation (14) provides the contribution of block j of unit i to the probability distribution function of unit i.

Probability Distribution Function of Generation Bus Power

For the probabilistic power flow analysis, of interest is the probability distribution function of the total generation at a bus. Consider generation bus k which comprises the set of generating units M(k). The total generation is denoted with the random variable Y_k. Thus,

$$Y_k = \sum_{i\in M(k)} X_i \qquad (15)$$

$$F_{Y_k}(y) = \Pr[Y_k < y] \qquad (16)$$

where $F_{Y_k}(y)$ is the probability distribution function of the variable Y_k. It is computed by summing up the contributions from all generation blocks belonging to units of bus k. For this purpose, consider the block j of unit i which is connected to bus k. The contribution of this block depends on the availability of the generating units of bus k already loaded. For purposes of explaining the pertinent equations, the following definitions are introduced:

M Set of generating units partially or fully loaded before block j, unit i

M' Subset of M comprising the generating units not connected to generation bus k

M" Subset of M comprising the generating units connected to generation bus k, excluding unit i

M(k) Set of generating units connected to bus k

$L_a = L - \sum_{\nu\in M'} X_\nu$ Equivalent load "seen" by the units of generation bus k

E_ℓ An event defined as a specific combination of available/unavailable units in the set M". Each event E_ℓ corresponds to generation z at bus k equal to

$$z = \sum_{\nu\in M''} x_\nu$$

Using the introduced notation, the contribution of block j of unit i to the function $F_{Y_k}(y)$ is computed with:

$$\Pr[Y_k<y] = \sum_\ell \Pr\left[\left(\sum_{\nu\in M''} X_\nu + X_i < y\right) | E_\ell E_i\right]\Pr[E_\ell]\Pr[E_i]$$

$$+ \sum_\ell \Pr\left[\left(\sum_{\nu\in M''} X_\nu + X_i < y\right) | E_\ell \overline{E}_i\right]\Pr[E_\ell]\Pr[\overline{E}_i] \qquad (17)$$

Note that:

$$\Pr\left[\left(\sum_{\nu\in M''} X_\nu + X_i < y\right) | E_\ell E_i\right] = F_{L_a}(z+\alpha)$$

$$\Pr[E_\ell] = \Pr\left[\sum_{\nu\in M''} X_\nu = z\right]$$

$$\Pr[E_i] = p_i$$

$$\Pr\left[\left(\sum_{\nu\in M''} X_\nu + X_i < y\right) | E_\ell \overline{E}_i\right] = F_{L_a}(z)$$

Upon mathematical manipulation and replacing the summation with integration yields:

$$\Pr[Y_k<y] = p_i \int_{z=0}^{y-\alpha} dF(z) \int_{\ell=0}^{\min(y,z+\alpha)} dF_{L_a}(\ell)$$

$$+ q_i \int_{z=0}^{y} dF(z) \int_{\ell=0}^{\min(y,z)} dF_{L_a}(\ell)$$

$$\alpha_j < \alpha < \alpha_{j+1} \qquad (18)$$

where:

F(z) is the cumulative probability function of the variable $z = \sum_{\nu\in M''} X_\nu$.

Equation (18) provides the contribution of block j, unit i, of bus k to the probability distribution function of Y_k. The integral (18) is computed for each block of all units connected to bus k. Upon completion, the probability distribution function of the total generation Y_k at bus k is known.

Probabilistic Power Flow Analysis

The simulation method described so far provides the description of power injections to system buses. Given this information, it is desirable to compute the probability distribution of circuit flows or bus voltage magnitudes. For this purpose, a power flow solution is obtained assuming the power injection at the system buses is equal to the expected values of power injections at the various buses. The expected values of power injections are computed from the calculated distributions defined with Eq. (16). Subsequently, a linearized model of circuit flows and bus voltages is developed in terms of power injections at generation buses. This linearized model includes the effects of electric load variation since electric load changes are absorbed by generation changes. Thus, in general, a circuit flow or a bus voltage magnitude, which is represented with a random variable W, is expressed as a linear combination of the power injections Y at the system generation buses:

$$W = \sum_{k=1}^{n} a_k(Y_k - \overline{Y}_k) \tag{19}$$

where:

a_k = Known constant coefficients
Y_k = Power injection at the kth generation bus
$\overline{Y}_k$ = Expected value of power injection at the kth generation bus.

The probability distribution of the random variable W is computed from the known probabilistic models of the power injections Y_k. As a matter of fact, the power injections Y_k are expressed as the sum of unit output at bus k, yielding:

$$W = \sum_{k=1}^{n} a_k \sum_{i \in M(k)} (X_i - \overline{X}_i) \tag{20}$$

where:

M(k) is the set of units connected to bus k
X_i is the output of unit i
$\overline{X}_i$ is the expected value of unit i output.

The probability distribution function of the random variable W is computed as a by-product of the simulation procedure described in the previous section. Specifically, for each loading of a block of a unit, the contribution to the probability density function of the variable W is computed. Consider, for example, the loading of block j, unit i illustrated in Fig. 2. Denote this event as follows:

$$E_2 = [\text{loading of block j, unit i}]$$

The probability of the event E_2 equals dp_{ji} illustrated in Fig. 2.

$$Pr[E_2] = dp_{ji}$$

The loading of block j, unit i contributes to the variable W by an amount equal to $a_k x_i$, $\alpha_j \le x_i \le \alpha_{j+1}$, where a_k is defined with Eq. (19) and α_j, α_{j+1} are defined in Fig. 2. The probability of this contribution is equal to $p_i dp_{ji}$, where p_i is the probability of availability of unit i and dp_{ji} is depicted in Fig. 2. In addition, block j, unit i contributes zero to the variable W with probability $1-p_i$. The computation of the contributions to the probability distribution function of the variable W is performed recursively in parallel with the simulation method described earlier.

In summary, the probabilistic power flow consists of solving a usual power flow problem assuming that the power injections at the system buses are the expected values $\overline{Y}_k$. Subsequently, circuit flows and bus voltage magnitudes are expressed as a linear combination of the power injections. The linearization requires one forward and back substitution [23].

Handling of Network Nonlinearities

The proposed method has been extended to account for nonlinearities resulting from the power system network model. For this purpose, the system electric load is partitioned into a number ζ of segments. As an example, consider the independent random variable v_1 of the electric load model. Assuming that $\zeta = 3$, the following events may be defined

$$C_i = \{0.33(i-1) < v_1 < 0.33i\}, \; i = 1,2,3$$

Each of the above events represents a range of electric load given by Eq. (5). On the other hand, each of the above events has a probability of occurrence:

$$Pr[C_i] = p_{ci}$$

Now the method described in this paper is applied conditionally upon each event C_i and the results added. Note that for each event C_i, a different network operating condition will be used for linearization. The overall procedure is illustrated in Fig. 3. In this way, nonlinearities resulting from network models are taken into account.

Monte Carlo Simulation

Validation of the proposed method with actual system measurements is very difficult if not impossible. A viable validation method is by means of Monte Carlo simulation. The Monte Carlo simulation is based on exactly the same models of electric load, generation, and transmission system as the proposed method. In this way, the results of the Monte Carlo method are directly comparable to the results of the proposed method. Note that in the Monte Carlo method, the number of trials must be large for meaningful results. This requirement hinders the applicability of the Monte Carlo method to large scale power systems. For this reason, the validation procedure has been limited to small size power systems.

The Monte Carlo simulation has been applied to the 24 bus IEEE Reliability Test System [22]. Tests have been performed to determine the number of trials required for meaningful results. The tests consisted of increasing the number of trials while observing the results. It has been observed that when the number of trials exceeded 5,000, no appreciable changes occur to the results. Based on these observations, all subsequent Monte Carlo simulations were performed with 10,000 trials.

Example Results

The 24 bus IEEE Reliability Test System (RTS) [22] has been used as the example system. No circuit outages were assumed. Generating unit data (capacities and forced outage rates) and electric load variations were assumed to be those defined in Ref. [22]. Generating unit cost data were modified. Specifically, quadratic cost coefficients were defined as illustrated in Table 1. The purpose of the modification was to accentuate the effects of the economic dispatch process.

The simulation of this system for a period of one year has been considered. For simplicity, unit maintenance has been neglected. The electric load specified for the RTS in Ref. [22] is a conforming

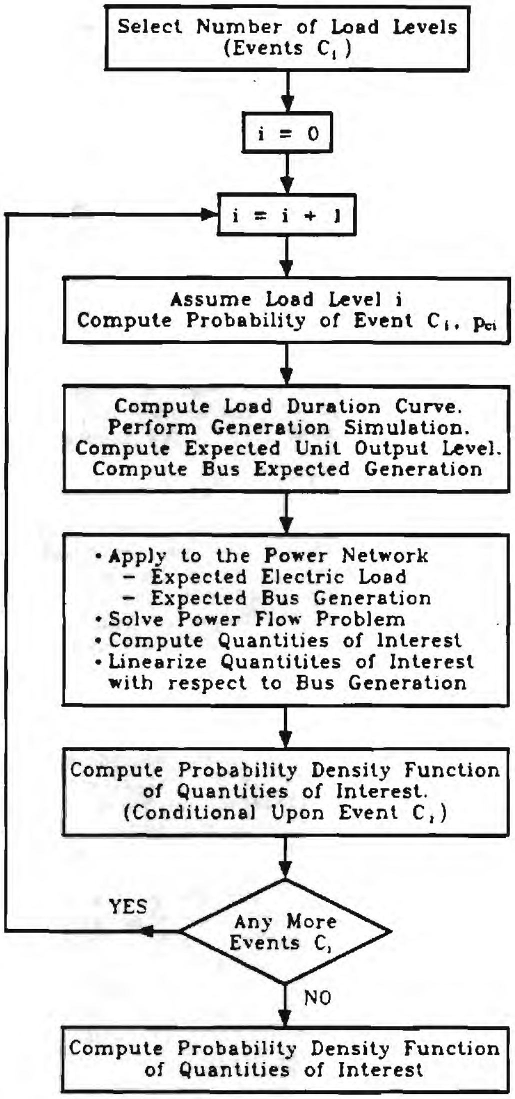

Figure 3. Flow Chart of the Proposed Probabilistic Power Flow Method.

load represented with a single independent variable. The parameters of the electric load model for a study period of one year have been computed as follows: First, the hourly chronological load data were constructed from the information proviced in Ref. [22]. These data are described with the equation:

$$\ell = 965.82 + 1884.3\ v\ (\text{MW})$$

where v is a variable assuming values between 0 and 1. Using above equation, the chronological load data ℓ were transformed into chronological data of the variable v. From these data, the probability distribution function of the variable v is computed. Subsequently, the total electric load is distributed to system buses (conforming load), yielding the following model of bus real power:

$$\begin{bmatrix} P1 \\ P2 \\ P3 \\ P4 \\ P5 \\ P6 \\ P7 \\ P8 \\ P9 \\ P10 \\ P13 \\ P14 \\ P15 \\ P16 \\ P18 \\ P19 \\ P20 \end{bmatrix} = \begin{bmatrix} 37 \\ 33 \\ 61 \\ 25 \\ 24 \\ 46 \\ 42 \\ 58 \\ 59 \\ 66 \\ 90 \\ 66 \\ 107 \\ 34 \\ 113 \\ 61 \\ 43 \end{bmatrix} + \begin{bmatrix} 71 \\ 64 \\ 119 \\ 49 \\ 47 \\ 90 \\ 83 \\ 113 \\ 116 \\ 129 \\ 175 \\ 128 \\ 209 \\ 66 \\ 220 \\ 120 \\ 85 \end{bmatrix} v$$

In addition, the complex power at a bus of the RTS system is:

$$S_i = P_i + j0.2P_i$$

Table 1. Generating Unit Data

Unit Size	# of Units	F.O.R.	Cost Coefficients a	b	c
12	5	0.02	14	32.0	0.01
20	4	0.10	1	45.0	0.001
50	6	0.01	0	0.1	0.011
76	4	0.02	87	16.0	0.02
100	3	0.04	98	26.0	0.01
155	4	0.04	124	13.0	0.015
197	3	0.05	113	24.0	0.01
350	1	0.08	180	24.0	0.014
400	2	0.12	229	7.0	0.016

Probability distribution functions of circuit flows and bus voltage magnitudes have been computed with the proposed method and with Monte Carlo simulation. Figures 4 and 5 illustrate typical results. Figure 4 includes the probability distribution of a circuit flow and a bus voltage magnitude computed using a single segment representation of the electric load. Note that the circuit flow matches reasonably well the Monte Carlo results, while the bus voltage magnitude shows substantial deviations from Monte Carlo results. The differences are attributed to the nonlinearities of the power flow equations. Figure 5 illustrates the distributions of flow and voltage magnitude for the same circuit and bus as in Fig. 4. The results of Fig. 5 were obtained by using a three segment representation of the electric load as it has been discussed earlier, applying the method for each load segment separately, and adding the results. Note that these distributions match the Monte Carlo results much better. In general, representing the electric load with more segments will provide better results. However, it should be observed that the amount of computation is proportional to the number of segments. Using three segments provides a good compromise between accuracy and speed. All the simulation results presented in Figs. 4 and 5 have been obtained with a step size of 5 MW for the incremental economic dispatch.

Evaluation of Method Efficiency

The proposed method consists of three basic computational procedures: (1) generation system simulation method, (2) standard power flow solution, and (3) linearization of quantities of interest such as bus voltage magnitude, circuit flows, etc.

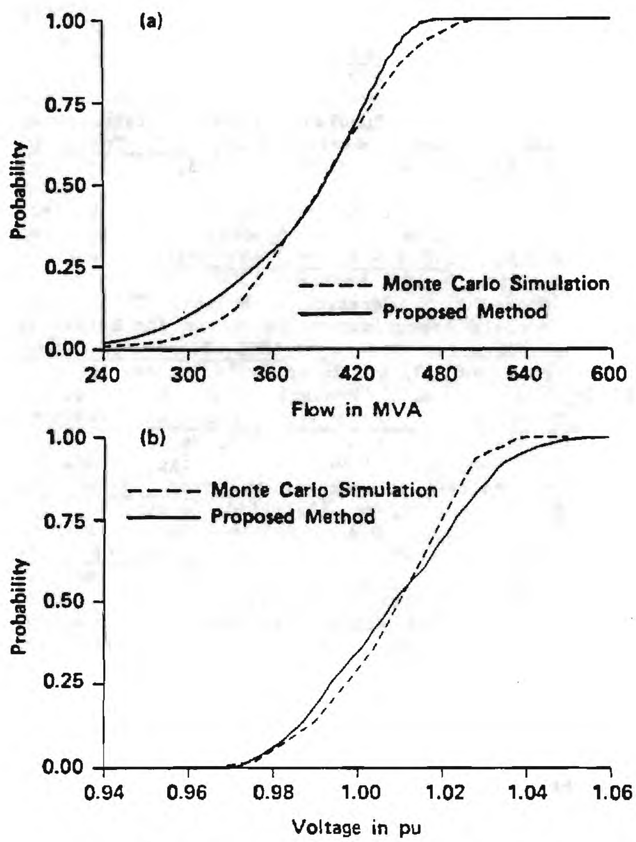

Figure 4. Probability Distribution Functions Using a Single Segment Load Model.
(a) Circuit 14–16 Flow
(b) Bus 6 Voltage

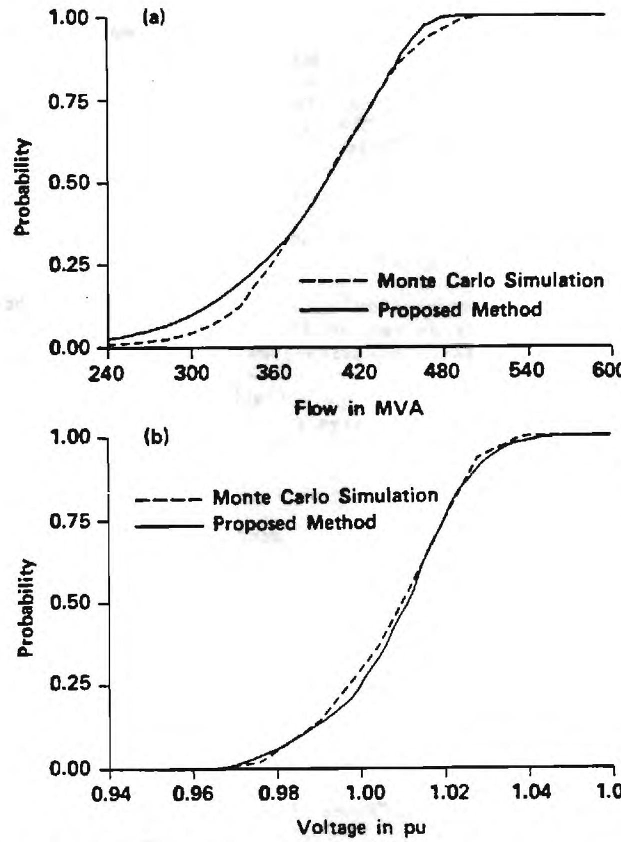

Figure 5. Probability Distribution Functions Using a Three Segment Load Model
(a) Circuit 14–16 Flow
(b) Bus 6 Voltage

The performance of standard power flow algorithms is well known and will not be discussed here. The linearization procedure is based on the costate method presented in Ref. [23]. The performance of this method has been well documented in [23]. Specifically, the computations required for the linearization of a specific quantity of interest (for example, a circuit flow) are approximately equal to that of a forward and back substitution with the table of factors of the Jacobian matrix of a standard Newton-Raphson power flow. To complete the performance evaluation of the proposed method, it is sufficient to provide data on the performance of the generation system simulation method. The execution time of the generation system simulation method depends on two main parameters: (1) number of units and (2) step size for simulating the incremental economic dispatch. Execution times of the method versus step size and parametrically with the number of units is given in Fig. 6. The results have been obtained with two systems: (1) a 32 unit system (the IEEE Reliability Test System) and (2) a 62 unit, 4198 MW peak load system. The results has been obtained on an IBM PS/2 Model 80, 20 MHz.

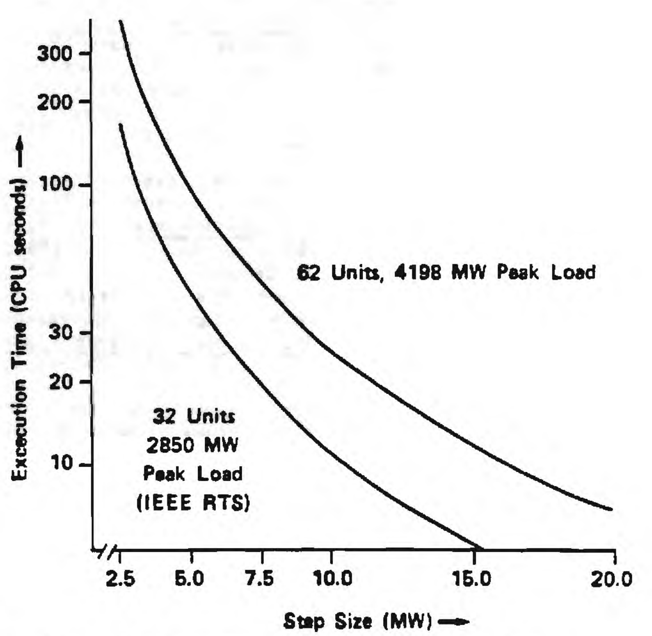

Figure 6. Execution Time of the Generation System Simulation Method. (IBM PS/2 Model 80, 20 MHz)

Conclusions

A new probabilistic power flow analysis method is proposed, capable of computing probability distribution functions of circuit flows and bus voltage magnitudes. The method is based on a description of bus power injections as random variables. The computation of the statistics of the bus power injection takes into consideration the major operating practices of power systems such as economic dispatch. Subsequently, circuit flows and bus voltage magnitudes are expressed as a linear combination of bus power injections. Their statistics are computed from the statistics of the power injections. The proposed method has been validated via Monte Carlo simulation.

The computational requirements of the method are moderate. Specifically, they are comparable to the sum of usual power flow analysis and probabilistic production costing [1].

The implementation of the method is straightforward. As a matter of fact it can be implemented with appropriate modifications of a power flow algorithm and a probabilistic production costing algorithm. Potential applications of the method are (1) reliability analysis of power systems, and (2) transmission loss evaluation.

Acknowledgements

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Biosketches

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APPENDIX B: Transmission Loss Evaluation Based on Probabilistic Power Flow

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TRANSMISSION LOSS EVALUATION BASED ON PROBABILISTIC POWER FLOW

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ABSTRACT

A simulation method of the composite power system is proposed for the purpose of evaluating the probability distribution function of transmission losses. The method accounts for uncertainty of the electric load, availability of generating units, nonlinearities in the power flow equations, and major operating practices. The method is based on the following procedure. First, given the probabilistic electric load model, the probability distribution function of the power injection at generation buses is computed by taking into consideration the availability of generating units and economic dispatch practices. Next, transmission losses are expressed as a piecewise linear function of power injections at generation buses. Subsequently, the probability distribution function of transmission losses is computed. Validation of the method is performed via Monte Carlo simulation. The method has been applied to the 24 bus IEEE reliability test system and the results are validated by comparing it to Monte Carlo simulation results. The method has also been applied to the Georgia Power Company's composite system (1304 buses, 98 units, 1546 lines, 117 transformers) and the results are presented. THe efficiency of the method is also documented with timing on the 1304 bus system.

KEY WORDS: Power flow, economic dispatch, stochastic load model, probability distribution function (PDF), PDF of transmission losses, Monte Carlo simulation.

1. INTRODUCTION

Accurate evaluation of transmission losses is important, both in planning studies and fair allocation of transmission loss cost among the members of an interconnected system. Transmission losses incur a capacity loss and an energy loss. Capacity loss is determined by the peak value of transmission losses while energy loss is determined by the average transmission loss.

Transmission loss is becoming an important factor as the operation of the electric power transmission is undergoing a transformation due to requirements for wheeling, independent power producers, cogeneration, load management programs, and other recent trends. These trends will increase the uncertainty associated with transmission loss evaluation and will incur a wider spread of transmission loss distribution over a specified time period [1].

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Transmission loss computation methods for economic dispatch or optimal power flows have been well developed [2-4]. In the classical economic dispatch approach, a quadratic expression for the losses is computed as a function of the generation schedule. This formula is considered constant. In optimal power flows the formula is updated at each iteration. As an approximation, capacity loss is computed as the transmission loss at peak load.

Energy loss is defined as the integral of losses over a specified period of time. Many decades ago, an approximate loss formula was developed which is still used in planning studies. This is the so-called loss factor method which is defined as follows: The energy loss, E_{loss}, is equal to:

$$E_{loss} = L_f \cdot P_{\ell peak} \cdot T$$

where

L_f : loss factor
$P_{\ell peak}$: peak power loss
T : time interval of interest.

If the loss factor in a system is exactly known, the above formula will provide the exact energy losses. However, the exact computation of the loss factor is a very difficult problem. An approximate expression has been developed many decades ago which relates the loss factor to the load factor as follows:

$$L_f = LF \cdot x + (LF)^2 \cdot (1-x)$$

where

LF = load factor = (average load/peak load)
x : a coefficient dependent on the system.

The coefficient x is typically between 0.3 and 0.15. The above formula has been originally developed for a radial system with distributed load which varies and has a load factor LF. Over the years, however, the assumptions of the loss factor method have been forgotten and the above loss factor expression has been used for network systems as well. Apparently, these methods provide only grossly approximate values for losses and have severe limitations.

The computation of peak and average transmission losses over a specified period of time requires a method which captures the multiplicity of possible operating conditions. To achieve this goal, two approaches can be employed: (1) the enumerative approach and (2) the probabilistic approach. In the enumerative approach, a number of highly probable operating conditions are determined, representing various load levels and possible generation system contingencies. Because the number of possible conditions is tremendous, one relies on experience and on system specific parameters to limit the number of operating conditions. As an example, this approach has been adopted by Georgia Power Company. Specifically, the four co-owners of the Georgia Integrated Transmission System (ITS), Georgia Power Company, Oglethorpe Power Corporation, Municipal Electric Authority of Georgia, and Dalton Utilities, commissioned a study to estimate transmission losses, peak and average. Estimated transmission losses are added to each co-owner's metered usage to calculate the gross usage. Power flow cases were run at the 40% through 100% load levels, at 10% intervals. The loads were uniformly scaled from the planning power flow base case. The generation was dispatched using the following simplification. All of the hydrogeneration and all thermal units with less than 500 megawatts net output were considered fully available for economic dispatch. The remaining thermal units were represented with their expected forced outage rates for peak (100% and 90% load levels) and expected availabilities for off-peak power flow runs. A weighted average transmission loss percentage was calculated at each of the seven load levels by assuming sibling unit outages to be identical (i.e. Bowen Unit 1 = Bowen Unit 2 outage, etc.) and then running 20 to 30 load flow cases at each of the seven load levels. The average transmission loss factor was determined by combining the loss data with a normalized load shape with 8760 hours of data points. This method provides acceptable results. The drawbacks of the method are: First, as the number of power interchange contracts increases and in view of future wheeling requirements, independent power producers, cogeneration, and load management programs, the number of probable operating conditions increases exponentially. Second, the confidence level of the method is low since, for practical reasons, only a tiny portion of all possible operating conditions is considered.

These drawbacks can be overcome by using a probabilistic approach based on probabilistic power flow formulations. Since the introduction of the probabilistic power flow notion [5], much work and improvements have been made in this area. Specific improvements relate to modeling of operating practices (mainly economic dispatch), representation of bus load correlation, handling the nonlinearities of the power flow equations and representation of probability distribution functions. Sauer and Heydt proposed the use of higher moments for improving the accuracy of the computed probability distribution functions [6]. da Silva, et al. [8], proposed a linear relationship between bus electric loads, thus assuming a fully correlated electric load model. While this assumption does not handle the general case, it is a step in the right direction to deal with electric load modeling issues. Allan and da Silva [9] introduced the concept of multilinearizations to deal with nonlinearities resulting from the power flow equations. Recently, the authors [23] have introduced a general electric load model based on a small number of independent random variables (principle components) and a rigorous formulation of the probabilistic power flow which accounts for important power system operating practices such as economic dispatch, unit forced outages, electric load uncertainty, and effects of nonlinearities in the power flow equations. This paper presents an application of this method for the purpose of computing the probability distribution function of transmission losses.

2. MODEL DESCRIPTION

The model described in this paper is based on the following assumptions:

(1) A probabilistic electric load model is given for a time interval T of interest.
(2) The generating unit parameters and forced outage rates are known.
(3) The transmission system is known.

Assumption (1) and (2) are general and nonrestrictive. Assumption (3) excludes transmission system contingencies. This assumption was introduced for two reasons: first, it is believed that the effect of transmission line outages on the distribution of losses is minor because of the short duration of line outages and, second, line outages may introduce severe nonlinearities which must be dealt with. Thus, in case transmission system contingencies must be considered the method should be applied to each specified contingency.

With the stated assumptions, the computation of the probability distribution function of transmission losses q, follows the following procedure. First, the electric load and generating system model is used to characterize the power generation Y, at system buses as random variables. The random variables, Y, are in general correlated. Subsequently, a probabilistic power flow is solved to yield the probability distribution function of q from the probabilistic model of the injections Y.

2.1 The Probabilistic Electric Load Model

The electric load model provides a probabilistic description of system bus loads and allows modeling of conforming or nonconforming bus loads. The assumptions of the probabilistic electric load model are: (1) Bus electric loads are typically strongly correlated. It is, therefore, reasonable to assume that they are generated as a linear combination of a small number of independent stochastic processes. (2) The power factor of the electric load at a specific bus is constant. The equations describing the electric load model for a system with n buses are:

$$\phi_1(B)z(t) = \phi_2(B)n(t) \tag{1}$$

$$z(t) = \phi_3(B)v(t) \tag{2}$$

$$P(t) = P_o + Av(t) \tag{3}$$

$$S_i(t) = P_i(t) + j\alpha_i P_i(t) \tag{4}$$

where

$n(t)$	is an m-vector of independent white noise processes
$z(t)$	is an m-vector of stationary stochastic processes
$v(t)$	is an m-vector of nonstationary stochastic processes
$P(t)$	is a vector of bus electric loads (real power)
ϕ_1,ϕ_2,ϕ_3	are vector functions of arbitrary polynomials
B	is the backward operator
P_o	is a constant n x 1 vector
A	is an n x m matrix
$S_i(t)$	is the complex electric load at bus i
α_i	is a constant for bus i; it is dependent upon the power factor of the load at bus i.

The model described with Eqs. (1) and (2) (ARIMA model) has been extensively used to represent the electric load. It is well known that it is capable of representing the periodicities as well as the nonstationary property of the electric load.

The total electric load is the summation of all bus loads. For a specified simulation time, it can be written as

$$L = \sum_{i=1}^{n} P_i(t) = a_o + \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}v_j \tag{5}$$

where

a_o	sum of elements in P_o
v_j	jth element in vector v(t) which is assumed to be distributed within (0,1).

The statistics of the random variables v can be obtained from the ARIMA model (1) and (2). From the known statistics of v, the probability distribution function, $F_L(\ell)$ of the total electric load L, are computed. The complementary distribution function, $L_o(\ell)$, of the total load is defined with

$$L_o(\ell) = 1.0 - F_L(\ell) = Pr[L > \ell] \qquad (6)$$

The complementary distribution function, $L_o(\ell)$, depends on m independent random variables v_i, $i = 1,2,\ldots,m$. The variables v_i are the independent principle components of the electric load. It should be pointed out that this model is a generalization of the load duration curve used in generation system reliability analysis. The functions $L_o(\ell)$ and $F_L(\ell)$ are describing the same thing, the statistics of the variable L.

2.2 The Generation System Model

A generating unit, i, of capacity c_i MW is modeled with a set of capacity states, each state with a specified probability. The probability density function of this model is expressed with

$$f_{X_i}(x_i) = \sum_{k=0}^{m_i} p_{ik}\delta(x_i - d_{ik}) \qquad (7)$$

where

m_i is the number of capacity states (including zero and full capacity of the unit)

d_{ik} are the capacities of the states (note, $d_{io} = 0.0$ and $d_{im_i} = c_i$).

For $m_i = 1$, this model is the well known two state (up and down) Markov model. For clarity of presentation, the method is discussed in terms of the up and down model of a unit. Unit outages are independent. In addition to this model, each unit is described with a production cost function versus unit output. This function can be a quadratic function or a piecewise linear function.

2.3 Simulation Method

Given the probabilistic electric load model for the time period under consideration and the list of available generating units, the operation of the system is simulated with probabilistic power flow method [23] to compute the probability distribution functions of the bus power injections and their correlations. This process accounts for the effects of economic scheduling function and the random forced outages of generating units.

Many refinements have been done to the classical probabilistic method which deals with the computation of probability of unit operation, expected energy produced by this unit, and expected cost of the unit [10-12]. Works include the speeding up of computer implementation [7,13,14] and improving the simulation method of operation practices such as economic dispatch, maintenance, algorithmic improvements [8,15-17] and improved representations of the duty cycle of peaking and midrange units [18-20]. In the following, a simplified description of the probabilistic simulation adopted from Ref. 15 is presented. Consider n units of the system operating at levels $x_1,x_2,\ldots,x_n$. If unit k is not in operation, then obviously x_k will equal 0. Since there is a finite probability that any unit can be forced out, the output of unit i, x_i, can be considered to be a random variable with probability of unavailability equal to q_i. We write

$$Pr(X_i = x_i) = 1 - q_i\,, \quad x_i \neq 0 \qquad (8)$$

$$Pr(X_i = 0) = q_i \qquad (9)$$

where X_i is a random variable representing the generation of unit i. Assume that the electric load equals ℓ. For this condition, the apparent load ℓ_a will be

$$\ell_a = \ell - x_1 - x_2 - \ldots - x_n \qquad (10)$$

Since $\ell,x_1,\ldots,x_n$ are not deterministically known, the above equation can be replaced with its equivalent equation in terms of the corresponding random variables

$$L_a = L - X_1 - X_2 - \ldots - X_n \qquad (11)$$

where L is a random variable representing the electric load and X_i is a random variable representing the output of unit i. Since the probability distribution functions of the random variables L, $X_1,\ldots,X_n$ are known and since these random variables are independent, the probability distribution function of the random variable L_a is computed with a series of convolutions.

If we assume that $\ell_a > 0$ (that is, load exceeds generation), then another unit should be brought into operation or one or more of the operating units should increase their output. Assume that unit i is operating at x_i and that it is selected according to a criterion to respond to any increases in the load. When the criterion is selected to be the incremental production cost of the unit, then the described procedure simulates the economic dispatch practice. In general, if $\ell_a > 0$, the output of unit i will increase from x_i to $x_i + \Delta x_i$, where Δx_i is a small increment (1-5 MW). We shall refer to this increment as the block Δx_i. It is noted that if $x_i = 0$, the increment Δx_i may not be small. In this case, unit i will be brought into operation at a level at least equal to minimum allowable operating level. With the described formulation and application of basic probability theory, the expected energy to be produced and cost of operation and required fuel are computed as follows:

Step 1: Compute the probability distribution function of random variable

$$L' = L_a + X_i$$

Let it be $F_{L'}(z)$. If $x_i = 0$, skip this step and assume $F_{L'}(z) = F_{L_a}(z)$. (Note that this step requires a deconvolution.)

Step 2: Compute

$$E(\Delta x_i) = (1-q_i)T \int_{z=x_i}^{x_i+\Delta x} \left(1-F_{L'}(z)\right)dz \qquad (12)$$

$$C(\Delta x_i) = (1-q_i)Tf_i(0)\left(1-F_{L'}(0)\right)\delta(x_i) + (1-q_i)T \int_{z=x_i}^{x_i+\Delta x_i} \frac{df(z)}{dz}\left(1-F_{L'}(z)\right)dz \qquad (13)$$

where

$\delta(x_i)$ = 1 if $x_i = 0$; $\delta(x_i) = 0$ if $x_i \neq 0$
$f(z)$ = production cost function of unit i
T = simulation time period (in hours)
$E(\Delta x_i)$ = simulation energy to be produced from block Δx_i
$C(\Delta x_i)$ = expected cost of operation of block Δx_i.

The above basic equations are utilized for the computation of the probability distribution function of bus total generation. Consider generation bus k which comprises the set of generating units M(k). The total generation is denoted with the random variable Y_k. Thus,

$$Y_k = \sum_{i\in M(k)} X_i \tag{14}$$

In Ref. 23 it is shown that the probability distribution function $Pr[Y_n \le y]$ of the total generation at bus k, Y_k is given by:

$$Pr[Y_k \le y] = p_i \int_{z=0}^{y-\alpha} dF(z) \int_{\ell=0}^{\min(y,z+\alpha)} dF_{L_a}(\ell) + q_i \int_{z=0}^{y} dF(z) \int_{\ell=0}^{\min(y,z)} dF_{L_a}(\ell)$$

$$\alpha_j \le \alpha \le \alpha_{j+1} \tag{15}$$

where:

$z = \sum_{\nu\in M^*} X_\nu$

M^* is the subset of M comprising the generating units connected to generation bus k

M is the set of generating units partially or fully loaded before block j, unit i

$F(z)$ is the cumulative probability function of the variable $z = \sum_{\nu\in M^*} X_\nu$.

Equation (15) provides the contribution of block j, unit i, of bus k to the probability distribution function of Y_k. The integral (15) is computed for each block of all units connected to bus k. Upon completion, the probability distribution function of the total generation Y_k at bus k is known.

2.4 Transmission Loss

The described simulation method has been extended to provide the probability distribution function of transmission loss. Specifically, the simulation method provides the description of power injections to system buses. Subsequently, a power flow solution is obtained assuming the power injection at the system buses is equal to the expected values of power injections at various buses. The expected values of bus generations are computed from the calculated distributions defined with Eq. (15). Next, the linearized model of transmission loss with respect to generation bus injections is computed as it is outlined in the Appendix, yielding:

$$q = q^o + \sum_{k=1}^{n} c_k\left(Y_k - \overline{Y}_k\right) \tag{16}$$

where

q^o is the transmission loss computed from a power flow assuming power injections equal to their expected values

$\overline{Y}_k$ is the expected value of total generation at bus k

Y_k is the total generation at bus k

c_k is the derivative dq/dY_k as it is outlined in the Appendix

n is the number of generation buses.

Let $W = q - q^o$. The probability distribution of the random variable W is computed from the known probabilistic models of the power injections Y_k. As a matter of fact, the power injections Y_k are expressed as the sum of unit output at bus k, yielding:

$$W = \sum_{k=1}^{n} c_k \sum_{i\in M(k)} \left(x_i - \overline{X}_i\right) \tag{17}$$

where

$M(k)$ is the set of units connected to bus k

X_i is the output of unit i

$\overline{X}_i$ is the expected value of unit i output.

The probability distribution function of the random variable W is computed as a by-product of the simulation procedure described in the previous section. Specifically, for each loading of a block of a unit, the contribution to the probability density function of the variable W is computed. Consider, for example, the loading of block j, unit i. Denote this event as follows:

$$E_2 = \text{[loading of block j, unit i]}$$

The probability of the event E_2 equals dp_{ji}.

$$Pr[E_2] = dp_{ji} \tag{18}$$

The loading of block j, unit i contributes to the variable W by an amount equal to $c_k x_i$, $\alpha \le x_i \le \alpha_{j+1}$, where c_k is defined with Eq. (16) and α_j, α_{j+1} are the limits of block j, unit i. The probability of this contribution is equal to $p_i dp_{ji}$, where p_i is the probability of availability of unit i. In addition, block j, unit i contributes zero to the variable W with probability $1-p_i$. The computation of the contributions to the probability distribution function of the variable W is performed recursively in parallel with the simulation method described earlier.

In summary, the probabilistic power flow consists of solving a usual power flow problem assuming that the power generations at the system buses are the expected values $\overline{Y}_k$. Subsequently, transmission losses are expressed as a linear combination of the power injections. The probability distribution function of transmission losses are computed using the simulation method.

3. MODELING OF NETWORK NONLINEARITIES

The proposed method has been extended to account for nonlinearities resulting from the power system network model. For this purpose, the system electric load is partitioned into a number ζ of segments. As an example, consider the independent random variable v_1 of the electric load model. Assuming that $\zeta = 3$, the following events may be defined:

$$C_i = \{0.33(i-1) \le v_1 \le 0.33i\}, \quad i = 1,2,3$$

Each of the above events represents a range of electric load given by Eq. (5). On the other hand, each of the above events has a probability of occurrence:

$$Pr[C_i] = p_{ci}$$

Now the method described in this paper is applied conditionally upon each event C_i and the results added. Note that for each event C_i, a different network operating condition will be used for linearization. The overall procedure is illustrated in Figure 1. In this way, nonlinearities resulting from network models are taken into account.

4. MONTE CARLO SIMULATION

Validation of the proposed method with actual system measurements is very difficult if not impossible. A viable validation method is by means of Monte Carlo simulation. For the validation procedure to be meaningful, the Monte Carlo simulation is based on exactly the same probability distribution of electric load generation, and transmission system availability as the one in the proposed method. In this way, the results of the Monte Carlo method are used as a benchmark. Note that the Monte Carlo simulation has been used only for validation purposes. No attempt has been made to determine optimal sample sizes or to

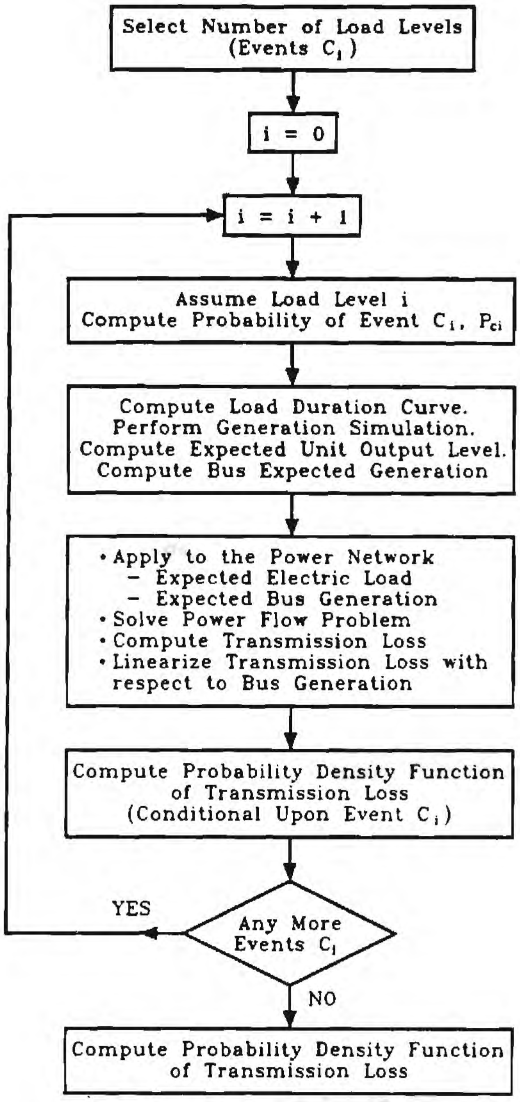

Figure 1. Flow Chart of the Proposed Method

reduce sample sizes by stratification techniques, etc. In any case, the number of trials must be large for meaningful results. This requirement hinders the applicability of the Monte Carlo method to large scale power systems. For this reason, the validation procedure has been limited to small size power systems.

The Monte Carlo simulation has been applied to the 24 bus IEEE Reliability Test System [21]. Tests have been performed to determine the number of trials required for meaningful results. The tests consisted of increasing the number of trials while observing the results. It has been observed that when the number of trials exceeded 5,000, no appreciable changes occur to the results. Based on these observations, all subsequent Monte Carlo simulations were performed with 10,000 trials. Limited Monte Carlo simulations have been also performed on the Georgia Power Company's bulk power system (1304 bus system).

5. SAMPLE RESULTS

The 24 bus IEEE Reliability Test System (RTS) [21] has been used as the example system. No circuit outages were assumed. Generating unit data (capacities and forced outage rates) and electric load variations were assumed to be those defined in Ref. 21. Generating unit cost data were modified. Specifically, quadratic cost coefficients were defined as illustrated in Table 1. The purpose of the modification was to accentuate the effects of the economic dispatch process.

Table 1. Generating Unit Data

Unit Size	# of Units	F.O.R.	Cost Coefficients a	b	c
12	5	0.02	13.70	33.51	0.455
20	4	0.10	0.68	56.00	0.375
50	6	0.01	86.76	17.92	0.0375
76	4	0.02	86.76	17.92	0.0375
100	3	0.04	97.03	28.72	0.0524
155	4	0.04	123.86	14.29	0.0126
197	3	0.05	112.44	25.26	0.0144
350	1	0.08	179.79	12.96	0.0026
400	2	0.12	228.31	6.50	0.0003

The simulation of this system for a period of one year has been considered. For simplicity, unit maintenance has been neglected. The electric load specified for the RTS in Ref. 21 is a conforming load represented with one independent variable (one principle component). The parameters of the electric load model for a study period of one year have been computed as follows: First, the hourly chronological load data were constructed from the information provided in Ref. 21. These data are described with the equation:

$$\ell = 965.82 + 1884.3\, v_1$$

where v_1 assumes values between 0 and 1. Using above equation, the chronological load data ℓ were transformed into chronological data of the variable v_1. From these data, the probability distribution function of the variable v_1 is computed. Subsequently, the total electric load is distributed to system buses (conforming load), yielding the following model of bus real power:

$$\begin{bmatrix} P1 \\ P2 \\ P3 \\ P4 \\ P5 \\ P6 \\ P7 \\ P8 \\ P9 \\ P10 \\ P13 \\ P14 \\ P15 \\ P16 \\ P18 \\ P19 \\ P20 \end{bmatrix} = \begin{bmatrix} 37 \\ 33 \\ 61 \\ 25 \\ 24 \\ 46 \\ 42 \\ 58 \\ 59 \\ 66 \\ 90 \\ 66 \\ 107 \\ 34 \\ 113 \\ 61 \\ 43 \end{bmatrix} + \begin{bmatrix} 71 \\ 64 \\ 119 \\ 49 \\ 47 \\ 90 \\ 83 \\ 113 \\ 116 \\ 129 \\ 175 \\ 128 \\ 209 \\ 66 \\ 220 \\ 120 \\ 85 \end{bmatrix} v_1$$

In addition, the complex power at a bus of the RTS system is:

$$S_i = P_i + j0.2P_i$$

The computed transmission loss probability distribution function for the IEEE-RTS is shown in Figure 2. Figure 2a illustrates the PDF as computed when the electric load is modeled with one segment only (solid line). The dashed line is the transmission loss PDF computed with Monte Carlo simulation. In the Monte

Carlo simulation, 10,000 trials have been performed of which 42 resulted in divergent power flows. The same information is illustrated in Figure 2b with the difference that the electric load is now represented with three segments. The Monte Carlo simulation results are identical as in 2a. Note that the results of the proposed method came closer to the Monte Carlo simulation as the electric load is represented with more segments.

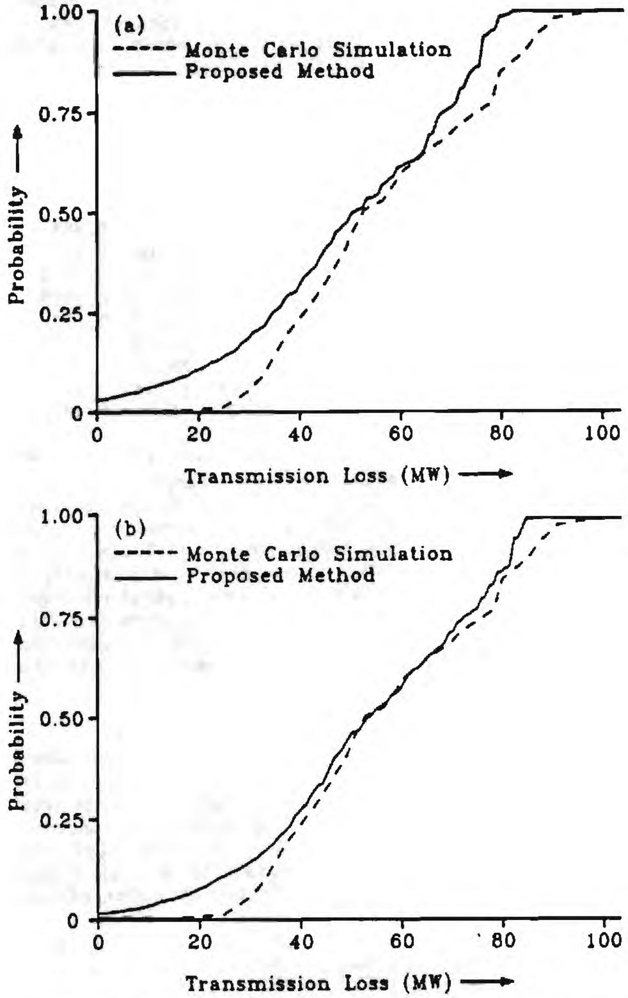

Figure 2. Transmission Loss Probability Distribution Function of the Reliability Test System
(a) One Segment Load Model
(b) Three Segment Load Model

The method has been also applied to the Georgia Power Company's bulk power system. The system model comprises 1304 buses, 98 units, 1546 lines, and 117 transformers. The obtained results are illustrated in Figure 3. The Monte Carlo simulation for this system consisted of 432 trials of which 32 resulted in diverged power flows. The limited number of trials is due to the enormous amount of computer time required to run a Monte Carlo simulation on a system of this size. Note that the agreement between Monte Carlo results and the proposed method is better for this system. Even when the electric load is represented with one segment, the agreement is good.

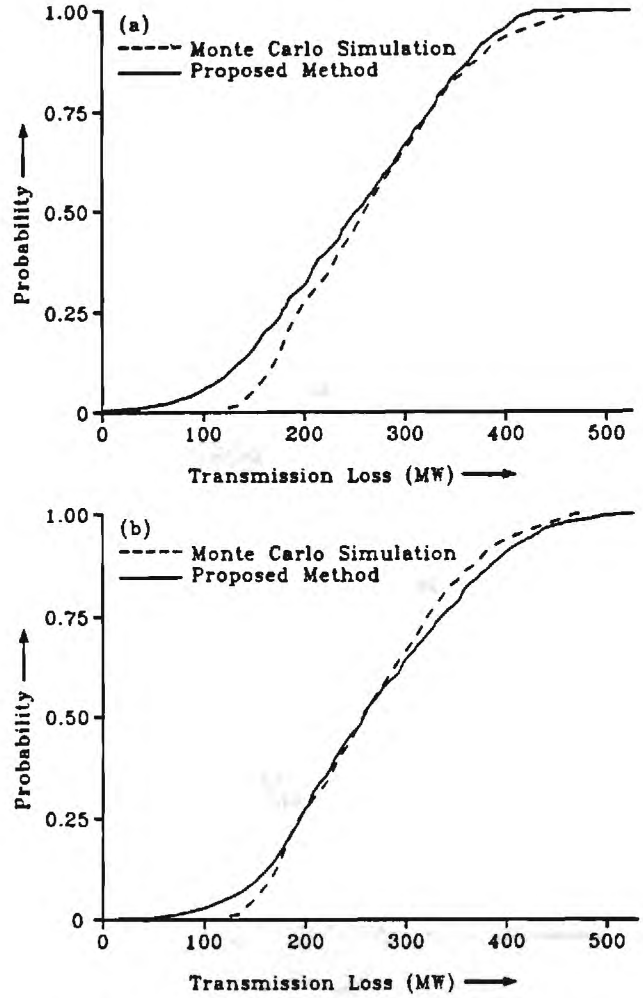

Figure 3. Transmission Loss Probability Distribution Function of the Georgia Power Company Bulk Power System
(a) One Segment Load Model
(b) Three Segment Load Model

The efficiency of the method is excellent as compared with Monte Carlo simulation. The analysis of the 1304 bus system was performed on an IBM PS/2 Model 70, 25 MHz personal computer. The recorded execution times for a 5 MW simulation step were as follows:

One Segment Electric Load:	8 mins, 33 secs
Three Segment Electric Load:	20 mins, 12 secs

For comparative purposes, the Monte Carlo simulation requires an average of 1 minute and 55 seconds per trial (power flow solution) on the same computer.

6. CONCLUSIONS

A probabilistic simulation method is proposed for the computation of the transmission loss probability distribution function. The method simulates the operation of the composite power system over a specified period of time. Electric load variations and unit availability uncertainty are explicitly modeled. Major operating practices, such as economic dispatch, are incorporated. The method has been validated using Monte Carlo simulation. The efficiency of the method, measured on a large power system, is excellent.

The transmission loss probability distribution function can be used to compute capacity loss (maximum value of transmission loss) and energy loss (expected value of transmission loss times period of interest).

ACKNOWLEDGEMENTS

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APPENDIX

This appendix presents the computation of the linearized loss equation (16).

Assume an operating point of the power system is defined with the variables V and the generation schedule Y. A change in one of the variables v, Δv_j, causes an incremental load change ΔL. Suppose the load change ΔL is absorbed only by one generation bus in the system, k, while any changes of power system transmission losses are taken care of by the system slack bus generation. Thus, a change Δv_j will bring a change in ΔY_k or vice versa. Given this definition, the transmission losses are a function of the variables ΔY_k. Upon expansion of the transmission loss formula in a McLaurin series in terms of ΔY_k and neglecting the higher order terms:

$$q = q^o + \sum_{k=1}^{n} c_k \Delta Y_k \tag{A-1}$$

where

$$c_k = \frac{dq}{dY_k} \tag{A-2}$$

The linearization coefficients c_k are computed as follows:

The transmission loss q is expressed as a linear combination of the power injections at all buses. Mathematically, the function q is expanded in a Taylor series as follows ignoring the high order terms:

$$q = q^o + \sum_{k=1}^{n} b_k \Delta\phi_k \tag{A-3}$$

where

$$\phi_k = Y_k - P_k = Y_k - P_{ok} - \sum_{j=1}^{m} a_{kj} V_j ,$$

real power injection at bus k

b_k = coefficients of linear term in Taylor series expansion
Y_k = power generation at bus k.

The total load change due to the variation of the independent variables V(t) can be written as (Eq. (5)):

$$\Delta L = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} \Delta V_j \qquad \text{(A-4)}$$

Assuming that this load increment is absorbed by one generation bus only, then:

$$\Delta Y_k = \Delta L = \sum_i \sum_j a_{ij} \Delta v_j \qquad \text{(A-5)}$$

In addition, any changes in transmission losses due to the increase of electric load and variable Y_k are absorbed by the slack bus. For the sake of simplicity, let $\Delta v_j = 0$ for $j = 1,\ldots,m$ except $j = \ell$. Under these conditions, power injections at the system buses is

$$\phi_k = Y_k - P_{ok} - a_{k\ell} v_\ell$$

or

$$\Delta\phi_k = \Delta Y_k - a_{k\ell} \Delta v_\ell$$

Substituting this equation, together with Eq. (A-5) back into Eq. (A-3) yields the model equation:

$$\begin{aligned} q &= q^o + \sum_k (b_k \Delta Y_k - b_k a_{k\ell} \Delta V_\ell) \\ &= q^o + \sum_k b_k \Big(1 - \frac{a_{k\ell}}{\sum_i a_{i\ell}}\Big) \Delta Y_k \\ &= q^o + \sum_k c_k \Delta Y_k \end{aligned} \qquad \text{(A-6)}$$

where

$$c_k = \frac{dq}{dY_k} = \Big(1 - \frac{a_{k\ell}}{\sum_i a_{i\ell}}\Big) b_k \qquad \text{(A-7)}$$

Equation (A-7) is the sensitivity of transmission losses with respect to the variable Y_i.

The coefficients b_k in Eq. (A-7) are computed from:

$$b_k = \frac{dq}{d\phi_k} = \frac{\partial q}{\partial \phi_k} - \hat{x}^T \frac{\partial g}{\partial \phi_k} = -\hat{x}^T \frac{\partial g}{\partial \phi_k} = \hat{x}_{\nu_k} \qquad \text{(A-8)}$$

where

$\hat{x} = [\frac{\partial q}{\partial x} J^{-1}]^T$ is the costate of the considered power system, see Ref. 22
x is the state of the power system (voltage and magnitude phase)
g is the power flow equations of the system
J is the Jacobian matrix of the power flow equation
ν_k is the index for the power injection ϕ_k in the Jacobian matrix.

From above equation it is apparent that the linearization of transmission loss requires the following steps: (1) computation of the vector $\partial q/\partial x$ through a series of substitutions, (2) computation of the costate vector x through a forward and back substitution with the table of factors of the Jacobian. Subsequently, b_i is obtained from (A-8) and c_k is obtained by direct substitution Eq. (A-7).

BIOSKETCHES

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APPENDIX C: Corrective Control For Voltage Security

A. P. Sakis Meliopoulos and Carol Cheng, 'Corrective Control For Voltage Security' presented at the NSF workshop on Bulk Power System Voltage Phenomena: Voltage Stability and Security, Potosi, Missouri, September 18-24, 1988.

CORRECTIVE CONTROL FOR VOLTAGE SECURITY

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ABSTRACT

This paper proposes a methodology for corrective control strategies which enhance voltage security. Voltage security encompasses present operating conditions and probable disturbances. For voltage security assessment, a new method is proposed which explicitly models the effects of voltage regulators. For corrective controls, a method is proposed which may alleviate or minimize voltage insecurity. Specifically, a problem formulation is proposed which leads to an optimal control problem. This problem is a large nonlinear optimization problem. A solution method is proposed based on a successive linear programming approach. The major innovations of the methodology are: (1) a model reduction method which is based on coherency analysis of problem constraints, and (2) a methodology to linearize the problem and to define the region of validity of the linearized model. This methodology allows the solution of this nonlinear model with successive linear programming approach.

The paper describes the overall method and provides typical results. Solution times of the proposed method are comparable to those of a power flow. Thus, the method is suitable for real time applications.

INTRODUCTION

Voltage security is becoming an increasing concern to both system operators and planners. There are two contributing factors for this trend. First, as power systems are being operated closer to their limits, voltage control problems are becoming of relevance. Second, analytical tools to deal with voltage security are not as well developed.

This paper addresses the analysis needs of voltage security and presents a new formulation. The new formulation quantifies voltage security and provides a mean for voltage security enhancement. The basic elements of the formulation are (1) a new method for contingency ranking which explicitly represents voltage regulators and model discontinuities, and (2) a corrective control strategy method to alleviate voltage problems.

VOLTAGE SECURITY ASSESSMENT METHOD

The voltage security assessment problem is posed as follows: Given a power system and the present operating conditions, determine whether the system will be capable to maintain the bus voltages within a desirable voltage range for the present conditions, as well as all possible single contingencies. Conceptually, the voltage security assessment method consists of analyzing all possible contingencies to determine any voltage security problems. This approach has been proven prohibitive.

An alternative approach is to a priori determine the contingencies which may lead to voltage insecurity (contingency ranking). Subsequently, only these contingencies are analyzed to determine the degree of voltage insecurity. Unfortunately, contingency ranking methods have not performed well for voltage problems because of the following factors: (1) inherent nonlinearities of voltage power equations, (2) available methods for contingency ranking do not address the effects of voltage regulators, (3) contingency ranking methods do not address the effects of discontinuities arising from limits on reactive power generation devices, and (4) the effects of voltage sensitive load are neglected.

A new method for contingency ranking is proposed based on a voltage sensitive performance index. The method introduces the effects of (1) load dependence on voltage, (2) effects of regulators in the PI ranking methods, and (3) effects of discontinuities when units reach their limit of reactive power generation. A brief description of the method is as follows:

The performance index is defined with

$$J = \Sigma w_i (V_i - V_i^{sp})^{2n} / (V_{imax} - V_{imin})^{2n} \tag{1}$$

where

V_i^{sp}	is the specified voltage for bus i
V_i	is the actual voltage at bus i
V_{imax}, V_{imin}	is the maximum and minimum allowable voltage at bus i
w_i	is a weight factor.

The power flow equations are symbolically expressed as

$$g(x,t,p) = 0 \tag{2}$$

where

- x is the vector of bus voltage phase and magnitudes
- t is the vector of tap ratios of voltage regulators
- p is the vector of contingency parameters, i.e., circuit impedance for circuit outages or unit power output for unit outages.

In addition, voltage regulators regulate the voltage at specific buses. This is written with

$$x_j - c_j = 0 \qquad \text{j index for voltage regulator} \tag{3}$$

Subsequently, we introduce the extended state vector x'

$$x' = \begin{bmatrix} x \\ t \end{bmatrix} \tag{4}$$

and the extended power flow equations

$$g'(x',p) = \begin{matrix} g(x,t,p) \\ x_j - c_j \end{matrix} = 0 \tag{5}$$

Contingency ranking is achieved by computing the change of the performance index J with respect to the contingency parameters:

$$\Delta J = \frac{dJ}{dp} \Delta p \tag{6}$$

where

$$\frac{dJ}{dp} = \frac{\partial J}{\partial p} - \hat{x}^T \frac{\partial g'(x',p)}{\partial p} \tag{7}$$

and

$$\hat{x}^T = \left(\frac{\partial J}{\partial x'}\right)^T \cdot \left(\frac{\partial g'(x',p)}{\partial x'}\right)^{-1} \tag{8}$$

It is noted that the term $\partial g'/\partial x'$ is the Jacobian matrix of the extended power flow equations. This Jacobian matrix consists of the usual Jacobian matrix plus some other terms resulting from the introduction of the regulator taps into the state vector and the regulation equations (3). The computation of the performance index change requires the computation of the modified Jacobian matrix and its table of factors, the computation of the vector $\partial J/\partial x'$, one forward and back substitution, and finally substitution into equation (7). Thus, the required computations are comparable to those of one iteration of the Newton-Raphson power flow.

The method accounts for the effects of voltage regulators. It is also capable of accounting for the effects of (1) load dependence on voltage and (2) effects of discontinuities when units reach their limit of reactive power generation. These items are discussed next.

A voltage sensitive electric load is represented as load consisting of two components: (1) a constant power load (independent of bus voltage) and (2) a constant impedance load. The decomposition of the load into these two components is done in such a way as to match the incremental dependency of the bus load on the voltage:

$$\frac{dP_{ti}}{dV_i} = 2g_i V_i \tag{9}$$

$$\frac{dQ_{ti}}{dV_i} = 2b_i V_i \tag{10}$$

where:

P_{ti}, Q_{ti} is the total bus load (real and reactive power)

g_i, b_i is the conductance and susceptance of the constant admittance load component of bus i load

V_i is the voltage of bus i at the operating point.

The parameters g_i and b_i contribute to the formation of the Jacobian matrix and, thus, provide the effect of the voltage dependent load to the computation of the contingency ranking equation (6).

Discontinuities resulting from limits on the reactive power capability of units are accounted as follows. When the units at a generation bus reach their reactive power capability limit, the bus voltage cannot be controlled. Instead, the reactive power injection is specified and the voltage is allowed to adjust. This condition changes the power flow equations (2) and, therefore, the Jacobian. Use of the proper Jacobian in Eq. (8) will account for the effect of this discontinuity in the contingency ranking criterion (6). Tests indicate that an effective way to account for these discontinuities is to produce two contingency ranking lists: (1) one assuming that the bus voltage at all generation buses is regulated and (2) one assuming that the bus voltage at all generation buses with reactive power margin below a threshold value is not regulated. This procedure is straightforward because it simply involves utilization of the contingency ranking method with two different Jacobian matrices.

VOLTAGE SECURITY CONTROLS METHOD

The problem of voltage security controls is posed as follows. Given a contingency with voltage problems, determine control actions which alleviate or minimize the voltage problems. This problem is solved with a linearized model which relates adjustments of control variables to low or high bus voltage and unit reactive power. The model is then utilized to solve for the required controls. The model allows for a variety of controls which are listed in Table 1. The overall method is iterative and it is illustrated in Figure 1. The figure indicates that, computationally, the methodology involves the following major steps:

1. Identification of failed operating constraints, low or high bus voltages, and generating unit reactive power output.
2. Analysis of coherent operating constraints (model reduction).
3. Selection and linearization of operating constraints (model set-up).
4. Formation and reduction of the LP model.
5. LP model solution.

TABLE 1. List of Available Controls for Voltage Security

- Generation Adjustment (real and reactive)
- Generation Bus Voltage Adjustment
- Transformer Tap Adjustment
- Switched Capacitors/Reactors
- Load Transfer
- Interchange Adjustments

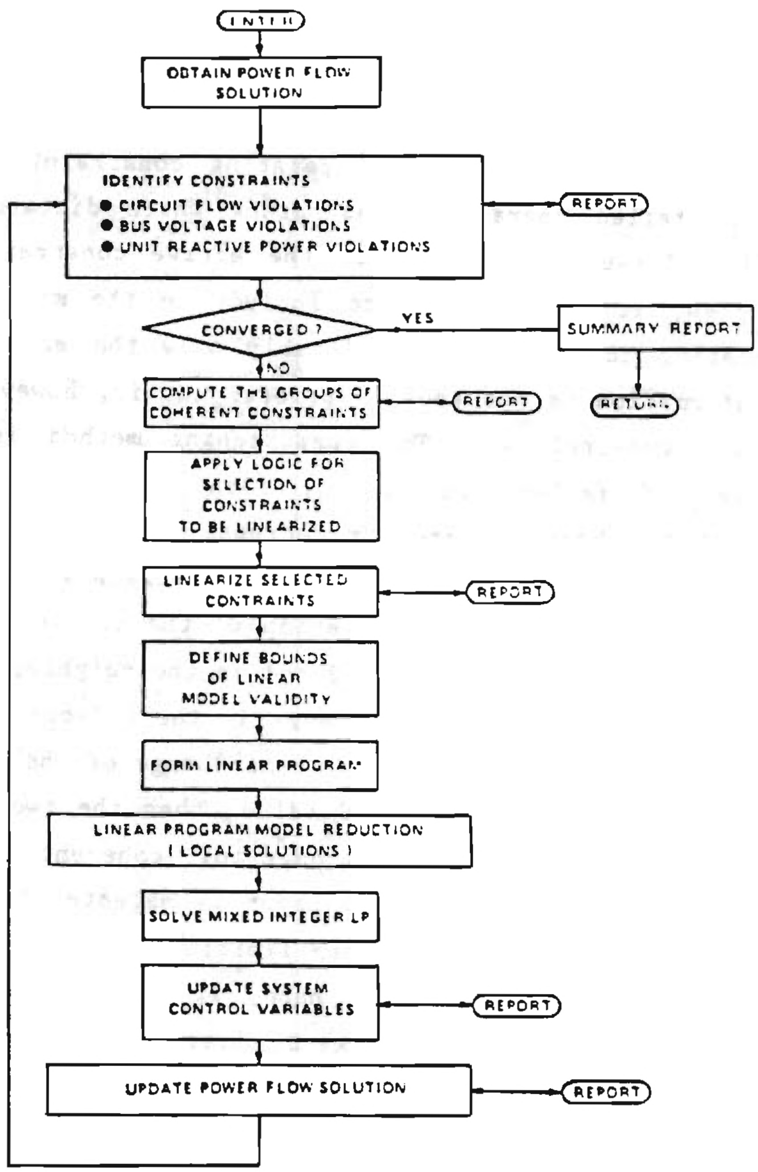

Figure 1. Flow Chart of the Voltage Security Controls Method.

6. System update.

These components of the method will be discussed next.

Identification of Failed Operating Constraints

This step is straightforward. Bus voltage magnitudes, and generating unit reactive power outputs are computed and the violations or near violations are identified and stored.

Analysis of Coherent Operating Constraints

For large power systems, the set of failed operating constraints may be large. However, the number of failed operating constraints which dictate the security controls may be small. These constraints are the active constraints. From the efficiency point of view, it is important to include in the model only a small number of failed operating constraints. Preferably only the active constraints. Unfortunately, this information is not known a priori. It is, however, possible to prescreen the operating constraints. The prescreening method is described as follows: Given a set of failed constraints, identify which constraints are "coherent," meaning that a subset increase or decrease together due to the application of security controls. An example of coherent constraints is the bus voltage magnitudes on a radial system. The identification of the coherent constraints is performed as follows: A set of controls is defined in the neighborhood of buses of the failed operating constraints. Then the change in the effective constraints is observed due to the defined controls. If the percent change of the violation of two constraints is within a prespecified threshold value, then the two constraints are classified as coherent. In this way, groups of coherent constraints are identified. The leading constraint from each group is selected to be included in the model. This procedure is very effective in limiting the number of constraints to be included in the model. On the other hand, it is not necessary that the constraint coherency analysis procedure be exact, i.e., to predict correctly all active constraints. Any wrong predictions will be identified and corrected later in the next iteration of the solution method.

Operating Constraint Linearization

Linearization of operating constraints requires the computation of sensitivities of the operating constraints with respect to the controls. Computationally, the procedure is the most demanding from all parts of the method.

The computation of sensitivities is done by direct differentiation of the quantity of interest. The resulting general expression of the sensitivity of a quantity f (bus voltage, unit reactive power, etc.) with respect to a control variable, u (transformer tap setting, bus voltage magnitude, etc.), is

$$\frac{df}{du} = \frac{\partial f}{\partial u} - \hat{x}^T \frac{\partial g}{\partial u} \tag{11}$$

where

f is the quantity of interest (constraint)

u is the control parameter of interest

g represents the power flow equations.

The vector x is defined as the solution of the equation

$$\hat{x}^T \left(\frac{\partial g}{\partial x}\right) = \left(\frac{\partial f}{\partial x}\right)^T \tag{12}$$

where

$\frac{\partial f}{\partial x}$ is the partial derivative of the quantity f with respect to the state variables x (bus voltage - phase and magnitude)

$\frac{\partial g}{\partial x}$ is the Jacobian matrix.

This formula was suggested by Dommel and Tinney [3]. In usual mathematical jargon, the vector $\hat{x}$ is called the costate of the system.

Solution Algorithm

The model is solved with an LP algorithm. The solution algorithm is described as follows: An optimization problem is defined:

Minimize: $J(x,u)$ (13)

Subject to: Power Balance Equation

$T_{ij} < \overline{T}_{ij}$, $ij \varepsilon I_T$ (Circuit Loading Constraints) (14)

$V_k < \overline{V}_k$ or $V_k > \underline{V}_k$, $k \varepsilon I_V$ (Voltage Constraints) (15)

$Q_\ell < \overline{Q}_\ell$ or $Q_\ell > \underline{Q}_\ell$, $\ell \varepsilon I_Q$ (Reactive Power Constraints) (16)

$$\underline{u}_m < u_m < \overline{u}_m \text{, all } m \qquad \text{(Limits on Controls)} \qquad (17)$$

where

x	is the system state (bus voltage phases and magnitudes)
u	is the control vector
T_{ij}	power flow on circuit ij
$\overline{T}_{ij}$	selected rating of circuit ij
I_T	is the set of selected circuit flow constraints to be included in the model
V_k	is the voltage magnitude at bus k
$\overline{V}_k, \underline{V}_k$	are upper and lower voltage limits at bus k
I_V	is the set of selected voltage constraints to be included in the model
Q_ℓ	is the reactive power generation at bus
$\overline{Q}_\ell, \underline{Q}_\ell$	are upper and lower reactive power limits at bus
I_Q	is the set of selected reactive power constraints to be included in the model
$\overline{u}, \underline{u}$	are upper and lower limits on the control variables.

The above optimization problem is linearized with the use of the sensitivity analysis. The procedure results in a large linear program in terms of the control variables u. Typically for a 1500 bus system, the number of variables in the linear program are in the order of several thousands. To increase efficiency, the size of the linear program is decreased (model reduction). The model reduction methodology developed is based on sensitivity information and does not affect the solution. A brief description of the method is as follows: Based on the sensitivity values, the control which is most effective to correct a failed constraint is identified. Next the remedial actions which have a sensitivity below a predetermined cutoff value (typically 0.1 of maximum sensitivity) are flagged as ineffective to correct the failed constraint. The procedure is repeated for all failed constraints. Then the controls which are ineffective for <u>all failed constraints</u> are eliminated from the model. It should be emphasized that the model reduction procedure does not affect the accuracy of the final result.

Validity of the Linearized Model

Since the method involves the linearization of a nonlinear problem, the validity of the linearized model must be addressed. Within the presented formulation, bounds on the control variables can be utilized to ensure that the solution remains within the region of validity of the linearized model. Specifically, the linearized model is valid in a neighborhood of the operating point at which the linearization is performed. Tests indicate that simple rules can be used to define the region of validity of the linearized model in terms of an acceptable error. For example, a simple rule to ensure validity of the linearized model is to limit bus VAR injection changes, Q, with the equation

$$\left|\frac{Q}{\Sigma Y}\right| < \alpha(e) \tag{18}$$

where

$\alpha(e)$ is an upper bound depending on the maximum acceptable error e; for example, $\alpha(e) = 0.01$

ΣY is the sum of all the admittances connected to the bus.

Another set of bounds is also defined in terms of the physical limitations on the control variables themselves. As an example, a transformer tap may be limited between 0.95 and 1.05. The bound utilized in the model is selected as the minimum of the two bounds mentioned above.

The importance of the bounds of the linearized model should be emphasized. Without the presence of these bounds, it is possible and most likely that the computed security controls will cause other constraints to be violated. In this sense, VAR limits of generating units are the most likely candidates. This will cause an oscillatory behavior of the solution method. The bounds prevent this from happening.

Discrete Remedial Actions

The complexity of the security controls method increases with the presence of discrete controls. Specifically, capacitor/reactor switching is performed in discrete steps and will be referred to as discrete security controls. A mathematically rigorous computational procedure for the best selection of discrete controls is complex and computationally expensive. For a practical solution, a suboptimal but efficient computational procedure for discrete controls is desirable. Such a procedure is described here.

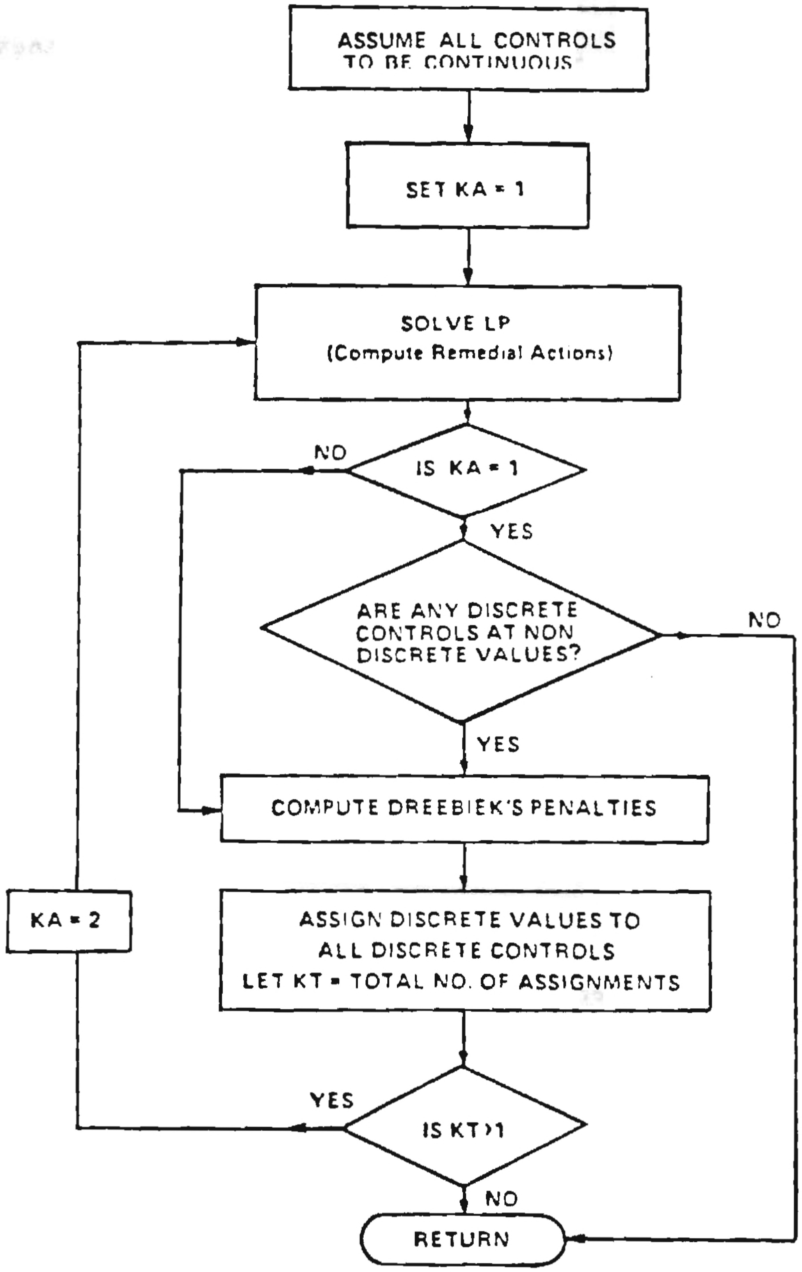

Figure 2. Algorithm for Discrete Controls.

A mixed integer linear programming approach is used for the calculation of discrete controls. The methodology is illustrated in Figures 1 and 2. Figure 1 illustrates the overall methodology for discrete controls. The solution of the mixed integer linear program is based on a suboptimal procedure using Dreebiek's penalties [8]. The method is illustrated in Figure 2. Note that Figure 2 is an expansion of the block designated as "SOLVE MIXED INTEGER LP" in Figure 1. Discrete controls cause large deviations of the operating conditions and, thus, may violate the range of validity of linearized models. Despite this, our studies indicate that in the case of capacitor/reactor switching, the linearized model can be effectively used to decide the preferred discrete value with reasonable accuracy. At the end, one additional load flow iteration is performed to verify the accuracy of the solution.

TEST RESULTS

The methodology described in this paper has been applied to the IEEE RTS (Reliability Test System) [15], which is illustrated in Figure 3. The system has been modified to allow the demonstration of the proposed method. Specifically, ten of the electric loads are assumed to be fed by ten voltage regulators as it is illustrated in Figure 3. Thus, the IEEE RTS system is now a 34 bus system. Other minor modification is that the transformer 3-24 is assumed to be a phase shifter and that the system is separated into three areas.

The contingency ranking method described in this paper has been applied to this system. Table 2 lists the 15 highest ranked circuit outages with the proposed method. The table provides the performance index gradient and the projected change of the performance index. The ranking of an outage is determined by the projected change of the performance index. For comparison purposes, the contingency ranking method without modeling voltage regulators has been also applied to the system and the results are listed in Table 2. Note that the rankings without the voltage regulator model are totally different. The superiority of the new method can be demonstrated as follows: For the first two highest ranked contingencies, the contingency power flow has been solved and the actual value of the performance index computed. The actual changes of the performance index for these contingencies are 15.85 and 12.60 versus 18.68 and 14.268 of the projected values. Note that these values are very close to each other. On the contrary, when the voltage regulators are neglected, the projected performance index change is 6.544 and 6.097, respectively, for these two contingencies, less than half of the actual change.

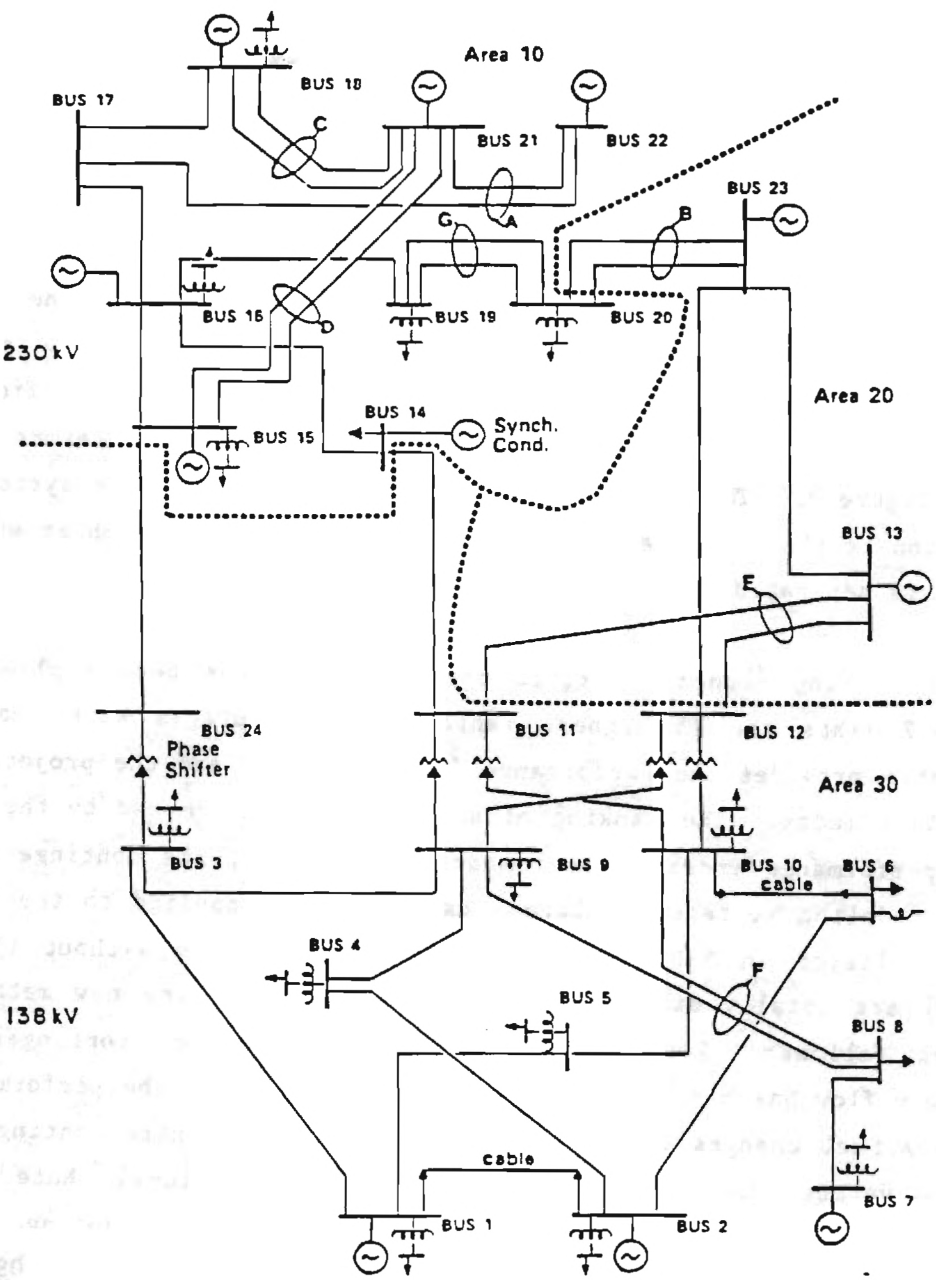

Figure 3. Modified IEEE Reliability Test System.

Table 2. Performance Comparison of Contingency Ranker

Circuit		With Regulator Model			Without Regulator MOdel		
From Bus	To Bus	Rank Ordr	PI Gradient	Proj. PI Change	Rank Ordr	PI Gradient	Proj. PI Change
9	11	1	1.568	18.680	4	.549	6.544
9	12	2	1.198	14.268	6	.512	6.097
11	14	3	.386	9.085	14	.079	1.862
12	13	4	.218	4.507	11	.156	3.226
14	16	5	.084	2.117	17	.052	1.305
12	23	6	.148	1.509	12	.265	2.697
10	12	7	.103	1.230	8	.408	4.862
8	9	8	.153	.869	19	.201	1.138
10	11	9	.050	.596	10	.392	4.673
1	5	10	.053	.585	21	.076	.838
16	17	11	.007	.249	23	.005	.173
15	16	12	.004	.248	25	.003	.143
5	10	13	.012	.131	24	.014	.150
17	22	14	.011	.102	28	.003	.026
20	23	15	.002	.072	35	-.001	-.065

Table 3. Voltage Security Controls for Circuit 9-11 Outage

Generator Bus Voltage Controls

Bus	Initial Value	Suggested value	Total Correction
1	1.0000	1.0077	0.0077
2	1.0000	1.0069	0.0069
7	1.0000	1.0500	0.0500
13	1.0100	1.0294	0.0194
15	0.9881	0.9981	0.0036
16	1.0000	1.0028	0.0028
23	1.0100	1.0500	0.0400

Transformer Tap Controls

Circuit	T. Buses	Initial Value	Suggested value	Total Correction
9	12	1.0000	1.0638	0.0638
10	11	1.0000	1.0753	0.0753
10	12	1.0000	0.9804	-0.0196

Transformer Phase Shift Controls

Circuit	T. Buses	Initial Value	Suggested value	Total Correction
3	24	0.00	-12.76	-12.76

Table 4. Security Constraint Analysis for Circuit 9-11 Outage

Bus Voltage Constraints

Bus	Initial Value	Final Value	Correction
3	0.926	0.949	0.023
5	0.934	0.959	0.025
12	0.941	0.985	0.044
14	0.922	0.963	0.041
13	0.949	0.949	0.000

Bus Reactive Power Constraints

Bus	Initial Value	Final Value	Correction
2	30.5	57.1	26.6

Table 5. Voltage Security Controls for Circuit 14-16 Outage

Generator Bus Voltage Controls

Bus	Initial Value	Suggested value	Total Correction
1	1.0000	1.0009	0.0009
2	1.0000	1.0037	0.0037
7	1.0000	1.0500	0.0500
13	1.0100	1.0037	-0.0063
15	0.9881	0.9901	0.0020
23	1.0100	1.0419	0.0319

Transformer Tap Controls

Circuit T. Buses		Initial Value	Suggested value	Total Correction
9	11	1.0000	1.0526	0.0526
9	12	1.0000	1.0638	0.0638
10	11	1.0000	1.0057	0.0057
10	12	1.0000	0.9940	-0.0060

Transformer Phase Shift Controls

Circuit T. Buses		Initial Value	Suggested value	Total Correction
3	24	0.00	-12.76	-12.76

Table 6. Security Constraint Analysis for Circuit 14-16 Outage

Bus Voltage Constraints

Bus	Initial Value	Final Value	Correction
3	0.915	0.936	0.021
5	0.943	0.960	0.017
12	0.945	0.980	0.035
14	0.940	0.971	0.031
34	0.949	0.960	0.011
9	0.939	0.951	0.012
19	0.942	0.950	0.008
6	1.050	1.050	0.000

Bus Reactive Power Constraints

Bus	Initial Value	Final Value	Correction
1	72.1	75.2	3.0
2	50.1	79.8	29.7
20	231.0	239.6	8.6

The security controls algorithm is successively applied to each contingency starting from the highest ranked contingency. As an example, the results of this algorithm on two contingencies are given, namely circuit 9-11 outage (rank 1) and circuit 14-16 outage (rank 5). The results are illustrated in Tables 3, 4, 5, and 6. Specifically, Table 3 lists the required controls to correct voltage problems under contingency 6-11. The analysis of the constraints for this case is illustrated in Table 4. The requirement is that all bus voltage magnitudes are in the range (0.95-1.05). Bus voltage constraints are satisfied. Note that a bus reactive power constraint is also listed. This is not a violated constraint; the reactive power capability at the bus is 80 MVAR. It is simply listed because during the iterative algorithm, this constraint became active. However, in the final solution, it is not active. The entire process required three major iterations (see Figure 1).

Table 5 lists the required controls to correct voltage problems under contingency 15-16. The analysis of the constraints for this case is illustrated in Table 6. All constraints are satisfied except the bus 3 voltage constraint (a value of 0.936 while the limit is 0.95). The reason for this is simply that the system does not have enough controls (excluding load shedding) to correct this voltage problem. Note also that these bus reactive power constraints are listed. These are not active constraints. During the solution at some point, they were active. This is the reason why these constraints are reported.

CONCLUSIONS

A new contingency ranking method is proposed which explicitly models voltage regulators. The method is easily implemented; it requires the introduction of an extended system state which, in turn, requires a modified Jacobian matrix in the application of the method. The performance of this method on the IEEE RTS is more superior than other methods. The contingency ranking method is combined with a voltage security controls algorithm which determines the controls required to correct voltage problems. The two algorithms together provide a comprehensive method for voltage security assessment and control.

ACKNOWLEDGEMENTS

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APPENDIX D: A New Contingency Ranking Method

A. P. Sakis Meliopoulos and Carol Cheng, 'A New Contingency Ranking Method,' Proceeding of the 1989 Southeastcon, Vol.2, pp.837-842, Columbia, South Carolina, April, 1989.

A NEW CONTINGENCY RANKING METHOD

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Abstract

A new contingency ranking method for detecting voltage problems is proposed. The method is based on the ac network model and a voltage sensitive performance index (PI). An efficient algorithm is proposed for the computation of the change of the performance index with respect to each contingency. The method accounts for the effects of (1) voltage regulators, (2) discontinuities arising from generation bus limits, (3) discontinuities arising from regulating transformer tap limits, and (4) voltage dependent loads. The concept of bus stiffness is introduced to provide an a priori knowledge of the accuracy of the ranking method. The computational requirements of the proposed method are comparable to that of one iteration of the Newton-Raphson power flow. The method is suitable for on-line voltage security assessment.

1. Introduction

Voltage security assessment is becoming an increasing concern to both system operators and planners. There are two contributing factors for this trend. First, as power systems are being operated closer to their limits, voltage control problems are becoming of relevance. Second, existing contingency screening/ranking methods for detecting voltage problems have not performed as well as those methods for circuit flow problems.

The problem of voltage security assessment can be briefly posed as follows: Given a power system and the present operating conditions, determine whether the system is capable of maintaining the bus voltage within a desirable voltage range for the present operating conditions, as well as for all possible single contingencies. Conceptually, the problem of voltage security assessment consists of simulating and analzying all possible contingencies so that any violations of the voltage limits can be detected. Computationally, this approach has been proven prohibitive. An alternative approach is to a priori determine the set of contingencies which may lead to voltage insecurity (contingency ranking). Subsequently, only these contingencies are analyzed to determine the degree of voltage insecurity. Unfortunately, present contingency ranking methods have not performed well for voltage problems because of the following factors: (1) misrankings due to the nonlinearities of reactive power equations, (2) existing methods neglect the effects of voltage regulators, (3) mistankings due to discontinuities arising from limits on reactive power generation devices, (4) misrankings due to discontinuities arising from limits on regulator taps, and (5) the effects of voltage sensitive load are typically neglected.

This paper proposes a new contingency ranking method based on the ac network model and a voltage sensitive performance index. The attributes of the method are as follows:

(1) Voltage regulators are explicitly represented in the contingency ranking algorithm.
(2) The method accounts for electric load sensitivity to bus voltage.
(3) Discontinuities arising from reactive power generation limits are incorporated in the method.
(4) Discontinuities arising from regulator tap limits are incorporated in the method.

The method also addresses the problem of misranking due to nonlinearities of the reactive power equations. This is achieved with the introduction of the concept of contingency stiffness.

The paper is organized as follows. First, the concept of contingency stiffness is discussed and its utilization to classify all possible contingencies into two groups. The first group of contingencies, which is the largest, can be effectively ranked with the proposed method which is described in subsequent sections. The second group of contingencies must be ranked with some other methods, such as subnetwork solutions, etc.

2. The Concept of Contingency Stiffness

One of the causes of misranking in performance index methods is the nonlinearity of the reactive power equations with respect to circuit or unit outage parameters. In a large power system, the degree of nonlinearity is dependent upon local network parameters. The degree of nonlinearity also determines whether the performance index method will correctly rank the outages. To determine the degree of nonlinearities involved in a specific outage, the concept of stiffness is introduced. This concept is illustrated with the aid of Fig. 1. The outage of circuit i causes a power unbalance at buses k and m by S_{km} and S_{mk}, respectively. The unbalance must be absorbed mainly by the circuits connected to buses k and m. The stiffness index is defined with

$$S_i = \max \left\{ \left|\frac{S_{km}}{Y_k^{eq}}\right| , \left|\frac{S_{mk}}{Y_m^{eq}}\right| \right\} \quad (1)$$

where:

Y_k^{eq} is the Norton equivalent admittance of bus k

Y_m^{eq} is the Norton equivalent admittance of bus m.

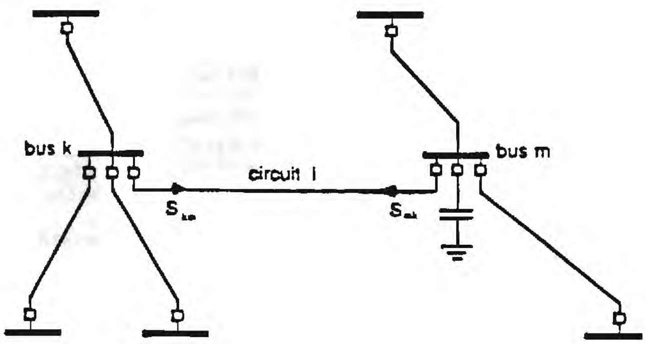

Figure 1 Illustration for the Definition of the Stiffness Index

The stiffness index defined with Eq. (1) provides a quantitative measure of the degree of local disturbance caused by the outage of circuit i. A small value of S_i means that the disturbance caused by the outage of circuit i is small. In this case, the system state and power flows change almost linearly with respect to the circuit i parameters. A linearized model of the power flow equations provides acceptable accuracy. Since the performance index method is based on a linearized model around the operating point, it is proposed that the stiffness index may be used as a criterion to determine which contingency can be well predicted by performance index ranking methods.

The definition of a stiffness index for generating unit outages is somehow more complex, since a unit outage may disturb the power balance in more than one bus. Let k1,k2,..., be the generator buses at which power injection changes will occur due to a unit outage at bus k. Let $p_1, p_2, \ldots$, be the participation factors for units 1,2,...,. The participation factor for a unit is defined in the usual sense of the economic dispatch problem. The stiffness index for a unit outage is defined with:

$$S_{k,g} = \max \left\{ \frac{P_k}{Y_k^{eq}}, \frac{p_1 P_k}{Y_{k1}^{eq}}, \frac{p_2 P_k}{Y_{k2}^{eq}}, \ldots \right\} \quad (2)$$

where P_k is the real power output of the unit prior to its outage. Again this stiffness index has the same physical meaning as the one defined for circuit outages.

The defined stiffness indices are applied as follows: For each contingency, the stiffness index is computed and compared to a threshold value. Contingencies with a stiffness index below a threshold value can be properly ranked with the method in this paper. Since the stiffness index is dependent only on the present operating condition, it can be computed in the preparation phase. The contingencies which do not fall in this category must be ranked with other methods such as subnetwork solutions, etc.

3. Description of the Method

This section provides a brief description of the method. The method is based on a performance index defined with

$$J = \sum_{k=1}^{N} W_k \left(\frac{2V_k - V_k^{max} - V_k^{min}}{V_k^{max} - V_k^{min}} \right)^{2n} \quad (3)$$

where:

V_k is the actual voltage magnitude at bus k

V_k^{max} is the maximum allowable voltage magnitude at bus k

V_k^{min} is the minimum allowable voltage magnitude at bus k

W_k is a weighting factor for bus k

n is an arbitrarily selected exponent.

The above defined performance index assumes a small value when all the voltage magnitudes are within the specified voltage limits and assumes a large value when one or more voltages are outside this range. Thus it provides a measure of voltage "normality" or security in the system. The ranking method involves the computation of the derivative of above performance index with respect to contingency parameters and subsequent ranking of the contingencies on the basis of the derivative values. Specifically, for every contingency j described with the parameter p_j, the first order approximation of the performance index change, ΔJ, is computed. Note that $\Delta J = (dJ/dp_j)\Delta p_j$. Contingencies are ranked on the basis of the values ΔJ. The major computational task for ΔJ is the computation of the derivative (dJ/dp_j). In subsequent paragraphs, the computation of the performance index derivative is illustrated.

3.1 Modeling of Voltage Regulators

Conventionally, the electric load of a power system is modeled as a constant power load. In practice, however, voltage regulators are used to maintain constant voltage at load buses, by changing the tap setting under load. In this case, the voltage at the regulated buses becomes a constant and the regulator tap becomes a variable. This condition is modeled with the power flow equations (4) and constraints (5) below:

$$g(x,t,p) = 0 \quad (4)$$

$$x_j - C_j = 0 \quad (5)$$

where:

- x is the state vector of bus voltage phase and magnitude
- t is the vector of tap ratios of voltage regulators
- p is the vector of contingency parameters. For circuit outage, it is the circuit impedance; for generating unit outage, it is the unit's power output.
- j is the index for voltage regulators
- x_j are voltage magnitudes at regulated buses.

We introduce the extended state vector x' defined as

$$x' = \binom{x}{t} \tag{6}$$

and the extended power flow equations

$$g'(x',p) = \begin{matrix} g(x,t,p) \\ x_j - C_j \end{matrix} = 0 \tag{7}$$

The derivative of the performance index is computed with the equation

$$\frac{dJ}{dp} = \frac{\partial J}{\partial p} - \hat{x}^T \frac{\partial g'(x',p)}{\partial p} \tag{8}$$

where $\hat{x}^T$ is the costate of the system, defined with

$$\hat{x}^T = \left(\frac{\partial J}{\partial x'}\right)^T \left(\frac{\partial g'(x',p)}{\partial x'}\right)^{-1} \tag{9}$$

In the equations above, the term $\partial g'/\partial x'$ is the Jacobian matrix of the extended power flow equations (7). It consists of the usual Jacobian matrix and some added entries resulting from the introduction of the regulator tap variables in the state vector and the regulation equations (5). The costate vector $\hat{x}$ is independent of the contingencies under consideration and needs to be computed only once. Thus, the computation of the derivative (8) requires the following steps:

(1) Formation and factorization of the modified Jacobian matrix.
(2) Computation of the vector $\partial J/\partial x'$.
(3) Computation of the costate vector $\hat{x}$ with one forward and back substitution (from Eq. (9)).
(4) Computation of the scalar $\partial J/\partial p$.
(5) Computation of the sparse vector $\partial g'(x',p)/\partial p$.
(6) Substitution into Eq. (8).

Steps (1), (2), and (3) need to be performed only once. Subsequently, for each contingency, Steps (4), (5), and (6) are performed to obtain the derivative dJ/dp. These computations are comparable to one iteration of the Newton-Raphson power flow.

3.2 Modeling of Voltage Dependent Loads

A voltage sensitive electric load is represented as a load consisting of two components: (1) a constant power load (independent of bus voltage), and (2) a constant impedance load. The decomposition of the load into these two components is done in such a way as to match the incremental dependency of the bus load on the voltage:

$$\frac{dP_{ti}}{dV_i} = 2g_i V_i \tag{10}$$

$$\frac{dQ_{ti}}{dV_i} = 2b_i V_i \tag{11}$$

where:

- P_{ti}, Q_{ti} is the total bus load (real and reactive power) at bus i
- g_i, b_i is the conductance and susceptance of the constant admittance load at bus i
- V_i is the voltage of bus i at the operating point.

3.3 Discontinuities Resulting from Reactive Power Capability Limits

Limits on reactive power generation capability cause discontinuities in the model of the power system. Specifically, when the units at a generation bus reach their reactive power limits, the bus voltage magnitude can not be controlled any more. Instead, the units operate at constant reactive power output and the voltage is allowed to adjust. In other words, a PV bus is converted into a PQ bus.

A certain contingency may cause the units at a bus reach their reactive power capability. Unfortunately, it is not known a priori what units may reach their limits. A procedure to retrieve this information is as follows. First, an appropriate performance index is defined

$$J_q = \sum_{k \in G} \frac{1}{2} \left(\frac{2Q_k - Q_k^{max} - Q_k^{min}}{Q_k^{max} - Q_k^{min}}\right)^{2n} \tag{12}$$

where:

- Q_k actual reactive power output at generation bus k
- Q_k^{max}, Q_k^{min} maximum and minimum reactive power capability of generation bus k
- G set of generation buses
- n an arbitrarily selected exponent.

The physical meaning of this performance index is as follows. For sufficient high values of the exponent n, when the value of J_p is greater than 1.0, the reactive power generation at one or more buses exceeds the allowable limits. Thus, by computing the J_q values for all contingencies, it is possible to identify the contingencies which cause one or more generation buses to hit their reactive power limits. The value of J_q for a contingency j is approximated with

$$J_q^{(j)} \simeq J_q^{(o)} + \frac{dJ_q}{dp_j} \Delta p_j \tag{13}$$

The derivative (dJ_p/dp_j) is computed with the same method as in Section 3.1. Thus, the procedure is equivalent to one iteration of the Newton-Raphson power flow method. The computed performance index values (13) are used as follows. The contingencies (j) which yield a value greater than 1.0 are ranked with the method of Section 3.1 by assuming that all generation buses are PQ buses. This means that in the application of the method only the Jacobian matrix will change. Use of the proper Jacobian matrix in the procedure of Section 3.1 accounts for the effects of discontinuities arising from reactive power capability limits.

3.4 Discontinuities Resulting from Regulator Tap Limits

Limits on tap settings for regulating transformers cause discontinuities in the model of the power system. Specifically, when a tap limit is reached (maximum or minimum), the regulating transformer can not control the voltage any longer. In this case, the transformer operates with a fixed tap (at maximum or minimum), thus becoming an off-nominal tap transformer. Again, the problem here is that it is not known a priori which contingencies may cause a regulating transformer to hit its tap limit. A procedure to predict this effect is by means of a performance index approach. Specifically, the following performance index is defined

$$J_t = \sum_{k\in R} \frac{1}{2}\left(\frac{2t_k - t_k^{max} - t_k^{min}}{t_k^{max} - t_k^{min}}\right)^{2n} \qquad (14)$$

where:

t_k	is the actual tap position of regulating transformer k
t_k^{max}, t_k^{min}	is the maximum and minimum tap setting of regulating transformer k
R	is the set of regulating transformers
n	is an arbitrarily defined integer.

The physical meaning of this performance index is as follows. For sufficient large values of n, when the value of J_t is greater than 1.0 the tap setting for one or more regulating transformers exceeds the limits. Thus, by computing the J_t values for all contingencies, it is possible to identify the contingencies which cause one or more regulating transformers to hit their tap limits. The values of J_t for a contingency j is approximated with

$$J_t^{(j)} = J_t^{(o)} + \frac{dJ_t}{dp_j}\Delta p_j \qquad (15)$$

The derivative (dJ_t/dp_j) is computed with the same method as in Section 3.1. Note again that, computationally, the above procedure is equivalent to one iteration of the Newton-Raphson power flow method. The computed performance index values (15) are used as follows: The contingencies (j) which yield a value greater than 1.0 are ranked with the method of Section 3.1 by assuming that all regulating transformers are off-nominal tap transformers. This means that in the application of the method, the usual Jacobian matrix must be employed. Use of this Jacobian matrix accounts for the fact that the regulating transformers reached its limits and operates as an off-nominal tap transformer.

4. Test Results

The methodology described in this paper has been applied to the IEEE RTS (Reliability Test System) [7], which is illustrated in Fig. 2. The system has been modified to allow the demonstration of the proposed method. Specifically, ten of the electric loads are assumed to be fed by ten voltage regulators as it is illustrated in Fig. 2. Thus, the IEEE RTS system is now a 34 bus sytem. Other minor modification is that the transformer 3-24 is assumed to be a phase shifter and that the system is separated into three areas.

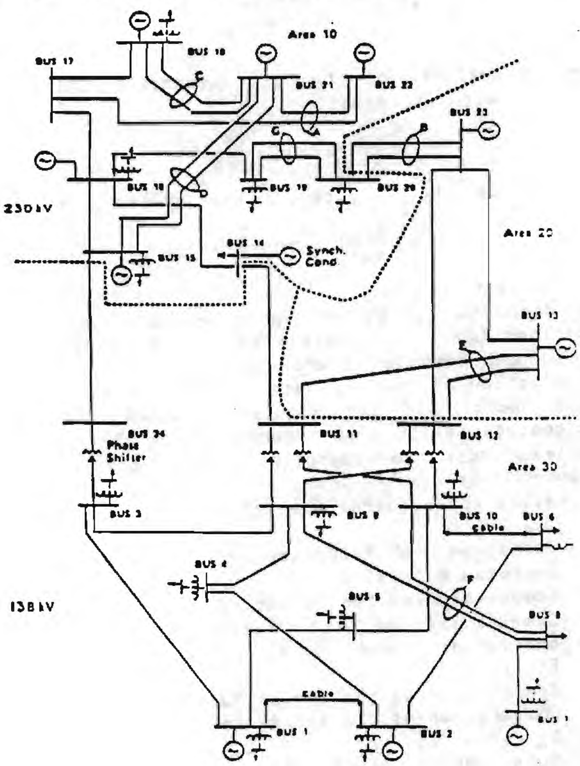

Figure 2 Modified IEEE Reliability Test System.

The contingency ranking method described in this paper has been applied to this system. Table 1 lists the 15 highest ranked circuit outages with the proposed method. The table provides the performance index gradient and the projected change of the performance index. The ranking of an outage is determined by the projected change of the performance index. For comparison purposes, the contingency ranking method without modeling voltage regulators has been also applied to the system and the results are listed in Table 1.

Table 1. Performance Comparison of Contingency Ranker

Circuit		With Regulator Model			Without Regulator Model		
From Bus	To Bus	Rank Ordr	PI Gradient	Proj. PI Change	Rank Ordr	PI Gradient	Proj. PI Change
9	11	1	1.568	18.680	4	.549	6.544
9	12	2	1.198	14.268	6	.512	6.097
11	14	3	.386	9.085	14	.079	1.862
12	13	4	.218	4.507	11	.156	3.226
14	16	5	.084	2.117	17	.052	1.305
12	23	6	.148	1.509	12	.265	2.697
10	12	7	.103	1.230	8	.408	4.862
8	9	8	.153	.869	19	.201	1.138
10	11	9	.050	.596	10	.392	4.673
1	5	10	.053	.585	21	.076	.838
16	17	11	.007	.249	23	.005	.173
15	16	12	.004	.248	25	.003	.143
5	10	13	.012	.131	24	.014	.150
17	22	14	.011	.102	28	.003	.026
20	23	15	.002	.072	35	-.001	-.065

Note that the rankings without the voltage regulator model are totally different. The superiority of the new method can be demonstrated by solving the contingency power flow, computing the actual value of the performance index, and comparing it to the projected value. As an example, the actual values of the performance index for the two highest ranked contingencies are 15.85 and 12.60 versus 18.68 and 14.268 of the projected values. Note that these values are very close to each other. On the contrary, when the voltage regulators are neglected, the projected performance index change is 6.544 and 6.097, respectively, for these two contingencies, less than half of the actual change. How well the method predicts the change in the performance index is also dependent upon the stiffness index. For example, for the outages 9-11 and 9-12, the stiffness index is 0.073 and 0.080, respectively. These values are relatively low and explain the good agreement between projected and actual performance index values. However, for contingency 14-16, the stiffness index is 0.198, a relatively high value. For this contingency the projected PI value is 2.117, while the actual value is 12.733. This contingency illustrates the usefulness of the stiffness index.

5. Conclusions

A new contingency ranking method is proposed which explicitly models voltage regulators. The method is easily implemented; it requires the introduction of an extended system state which, in turn, requires a modified Jacobian matrix in the application of the method. The innovations of the method are the following: (1) voltage regulators are explicitly modeled, (2) any form of the performance index can be accommodated, (3) discontinuities in the power system model arising from generating bus reactive power limits and regulator tap limits are effectively addressed, and (4) nonlinearities of the power flow equations with respect to contingency parameters are addressed by the introduction of the stiffness index. Preliminary results with the IEEE RTS system demonstrate the superiority of the method in predicting changes of the performance index value due to contingencies and, thus, correctly ranking contingencies. These tests also show a close correlation between the stiffness index and the validity of projected performance index changes. Thus the stiffness index, with respect to a contingency, can be used as a criterion to classify contingencies into two groups. The first group includes contingencies which can be properly ranked by the proposed method. The second group of contingencies must be ranked by other methods. Presently, methods are being investigated for ranking of the second group of contingencies. These methods are based on subnetwork solutions.

Acknowledgements

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APPENDIX E: A Hybrid Contingency Ranking Method

A. P. Sakis Meliopoulos and C. Cheng, 'A Hybrid Contingency Ranking Method,' Proceeding of the 10th Power System Computation Conference, pp 605-612, Graz, Austria, Aug. 1990.

A HYBRID CONTINGENCY RANKING METHOD

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ABSTRACT

A hybrid contingency ranking method for detecting voltage problems is proposed. The method is based on the AC network model of a power system. It employs an algorithm to classify contingencies into two groups. The first group comprises contingencies which can be effectively ranked with a performance index method, while the second group comprises contingencies which cause major nonlinearities. These contingencies can be effectively ranked with a subnetwork solution method. The method accounts for the effects of (1) voltage regulators, (2) discontinuities arising from generation reactive power limits, (3) discontinuities arising from regulating transformer tap limits, and (4) voltage dependent loads. The computational requirement of the proposed method are comparable to that of three iterations of the Newton-Raphson power flow.

INTRODUCTION

Contingency selection plays an important role in power system security assessment. Practically it is impossible to analyze all possible contingencies and thus the role of contingency selection is to select those contingencies which may have an adverse effect on system security.

An efficient approach to contingency selection is contingency ranking by means of a performance index (PI). A performance index is a heuristically defined scalar function of system parameters and state variables. Contingency ranking is the process of computing the changes of the performance index caused by each contingency, and ranking the contingencies in a descending order of performance index change.

Since the introduction of the concept of contingency selection [1], many efforts have been made in developing efficient methods to predict the severity of contingencies. PI ranking methods for detecting circuit overload problems have been intensively investigated [2-4]. These methods are based on the DC network model and are capable of computing second order changes of the performance index. The performance of these methods has been considered satisfactory in speed and accuracy, within the framework of the DC network model. However, the DC network model may not be satisfactory for many applications and it is not applicable to voltage security problems.

Recently, bounding methods have been developed based on a subnetwork solution [5,6,12]. They are capable of detecting local as well as system-wide circuit overload problems. Most recently, local methods have been generalized with the zero mismatch approach [18].

Performance Index (PI) methods for contingency ranking are extremely fast. Unfortunately, PI methods for voltage problems have not performed well because of the following factors: (1) misrankings due to the nonlinearities of reactive power equations, (2) omission of the effects of voltage regulators, (3) misrankings due to discontinuities arising from limits on generation reactive power, (4) misrankings due to discontinuities arising from limits on regulator taps, and (5) effects of voltage sensitive loads. On the other hand, subnetwork solution methods work well but they are substantially less efficient than PI methods.

The basic idea of the proposed hybrid ranking method is based on the observation that only a small percentage of contingencies cannot be accurately ranked with a PI method for the forementioned reasons. For these contingencies, a subnetwork solution method is used. The remaining contingencies, which represent the majority, can be effectively ranked with a comprehensive PI method [14,20], which accounts for the effects of voltage regulators and electric load sensitivity to bus voltage. This paper presents further developments of this method. Specifically, a two step method is proposed for the identification of contingencies which must be ranked with a subnetwork solution method. First, a performance index method is employed to predict which contingencies will cause discontinuities in the system model due to unit reactive power limits or transformer tap limits. These contingencies are ranked with a subnetwork solution method. Second, the concept of contingency stiffness is introduced to predict the severity of nonlinearities of the reactive power equations. Contingencies causing severe nonlinearities are also ranked with a subnetwork solution method. The remaining contingencies are ranked with a PI based method.

The paper is organized as follows. First the models and algorithms used to classify the contingencies into two groups are presented. Next, the performance index method is outlined. Finally, test results with three power systems are provided in the paper. The three systems are (1) the 24 bus IEEE Reliability Test System, (2) a 308 bus system of a Northeastern utility, and (3) a 1304 bus system of the Georgia Power Company. The results demonstrate the effectiveness of the method in predicting changes of the performance index due to contingencies and, thus, correctly ranking contingencies.

CLASSIFICATION OF CONTINGENCIES

In general, the causes of misrankings of PI methods are (1) nonlinearities and (2) discontinuities of the electric power system. It is very difficult to address these causes with a single performance index ranking algorithm. As an example, there are methods which partially account for nonlinearities, i.e. second order algorithms. However, the computational requirements and modeling restrictions, i.e. DC network model only, make these methods impractical. On the other hand, the number of contingencies that result in highly nonlinear system behavior or cause discontinuities in the system model are small compared with the total number of possible contingencies.

For these reasons, it is proposed that the total number of contingencies be classified into two groups: The first group consists of those contingencies that can be well predicted by a performance index ranking method. The second group consists of those that should be predicted by other methods.

We will refer to them as Class A and B contingencies, respectively. Class B contingencies are identified with an heuristic method based on the concept of contingency stiffness to be discussed next. Class A contingencies are the remaining contingencies. Subsequently, the identification of Class B contingencies is discussed.

Identification of Class B Contingencies

A major cause for misrankings of PI based contingency ranking methods is the nonlinearity of the power equations. In a large power system, the degree of nonlinearity caused by a contingency is dependent upon network parameters. To determine the degree of nonlinearities involved in a specific outage, the concept of contingency stiffness is introduced. This concept is explained with the aid of Figure 1. The outage of circuit i causes a power imbalance at buses k and m by S_{km} and S_{mk}, respectively. The imbalance must be absorbed mainly by the circuits connected to buses k and m. The stiffness index for the outage of circuit i is defined with

$$S_i = \max\left\{\left|\frac{S_{km}}{Y_k^{eq}}\right|, \left|\frac{S_{mk}}{Y_m^{eq}}\right|\right\} \tag{1}$$

where:

Y_k^{eq} is the equivalent admittance of bus k

Y_m^{eq} is the equivalent admittance of bus m.

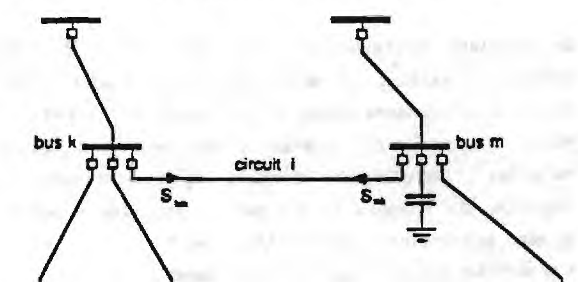

Figure 1 Illustration for the definition of the stiffness index

The stiffness index defined with (1) provides a quantitative measure of the degree of local disturbance caused by the outage of circuit i. A small value of S_i means that the disturbance caused by the outage of circuit i is small. In this case, the system state and power flow vary almost linearly with respect to the circuit i parameters. A linearized model of the power flow equations around the operating point provides acceptable accuracy. Since the performance index ranking method is based on such a model, it is proposed that the stiffness index may be used as a criterion to determine which contingency can be well predicted by performance index ranking methods.

The definition of a stiffness index for generating unit outages is somehow more complex, since a unit outage may disturb the power balance in more than one bus. Let k1,k2,..., be the generation buses at which power generation changes will occur due to a unit outage at bus k. Let $\Delta P_{k1}, \Delta P_{k2}, \ldots$, be the actual change of real power generations at these buses, which can be computed from ΔP_k, the real generation of the outaged unit, and the participation factors of other units. The stiffness index for a unit outage is defined with

$$S_{k,g} = \left\{ \max \frac{\Delta P_k}{Y_k^{eq}}, \frac{\Delta P_{k1}}{Y_{k1}^{eq}}, \frac{\Delta P_{k2}}{Y_{k2}^{eq}}, \ldots \right\} \tag{2}$$

Again this stiffness index has the same physical meaning as the one defined for circuit outages. Note that Eqs. (1) and (2) define contingency stiffness indices in terms of the maximum normalized disturbance caused to a bus due to the contingency.

The contingency stiffness indices are applied as follows: Contingencies with a stiffness index above a given threshold value are classified as Class B contingencies. The remaining are Class A contingencies.

CONTINGENCY RANKING ALGORITHM

This section presents the contingency ranking algorithm. Specifically, two ranking algorithms are employed: one for ranking class A contingencies and one for ranking class B contingencies. A brief description of the algorithms follows.

Ranking of Class A Contingencies

Class A contingencies are ranked with a first order PI method. As it has been mentioned, because of discontinuities, PI methods are prone to misranking. To alleviate this problem, a PI based method has been developed which predicts contingencies which cause discontinuities of the system model. A discussion of discontinuities resulting from generation bus reactive power limits and transformer tap limits is presented next.

Identification of Discontinuities: Generation Bus Reactive Power Limits

A contingency may cause the units of a bus to reach their reactive power generation limits. Unfortunately, it is not known a priori which units may reach limits. A procedure to retrieve this information is as follows. First, an appropriate performance index is defined as

$$J_q = \sum_{k \in G} \frac{1}{2} \left(\frac{Q_k - Q_{k,av}}{Q_{k,st}} \right)^{2n} \tag{3}$$

where:

Q_k : the actual reactive power output at generation bus k

$Q_{k,av}$ $= \frac{1}{2}(Q_k^{max} + Q_k^{min})$

$Q_{k,st}$ $= \frac{1}{2}(Q_k^{max} - Q_k^{min})$

Q_k^{max}, Q_k^{min} : the maximum and minimum reactive power capability of generation bus k

G : the set of generation buses

n : an arbitrarily defined integer.

The physical meaning of this performance index is as follows. For sufficient high values of the exponent, n, and for values of J_q greater than 1.0, the reactive power generation at one or more buses exceeds the allowable limits.

Next, a first order approximation of the value of J_q for a contingency j is computed with

$$J_q^{(j)} = J_q^{(o)} + \frac{dJ_q}{dp_j} \Delta p_j \tag{4}$$

where:

$J_q^{(o)}$: the performance index value at base case

p_j : the contingency parameter

$\frac{dJ_q}{dp_j}$: the derivative of J_q

The derivative $\frac{dJ_q}{dp_j}$ is computed with the method described in the next section. Once the values of $J_q^{(j)}$ for all contingencies are computed, the classification of contingencies is performed as follows: The contingencies which yield a $J_q^{(j)}$ value greater than 1.0 are analyzed with usual power flow analysis and excluded from the remaining procedure.

Identification of Discontinuities: Regulator Tap Limits

A contingency may cause a regulating transformer (TCUL) to hit the tap limit. It is not known a priori which contingencies may cause a regulating transformer to hit its tap limit and what will be the severity of this condition. A performance index J_t is defined to predict such cases.

$$J_t = \sum_{k\in R} \frac{1}{2}\left(\frac{t_k - t_{k,av}}{t_{k,st}}\right)^{2n} \tag{5}$$

where:

t_k : the actual tap position of regulating transformer k

$t_{k,av}$ $= \frac{1}{2}\,(t_k^{max} + t_k^{min})$

$t_{k,st}$ $= \frac{1}{2}\,(t_k^{max} - t_k^{min})$

t_k^{max}, t_k^{min} : the maximum and minimum tap setting of regulating transformer k

R : the set of regulating transformers

n : an arbitrarily defined integer.

For sufficient large values of n, and for values of J_t greater than 1.0, the tap setting of one or more regulating transformers exceeds the limits. Thus, by computing the J_t values for all contingencies, it is possible to identify the contingencies which cause regulating transformers to hit their tap limits. A first order approximation of the value of J_t for a contingency j is given with

$$J_t^{(j)} = J_t^{(o)} + \frac{dJ_t}{dp_j}\,\Delta p_j \tag{6}$$

where:

$J_t^{(j)}$: the performance index value at base case

p_j : the contingency parameter

$\frac{dJ_t}{dp_j}$: the derivative of J_t.

The derivative $\frac{dJ_t}{dp_j}$ is computed with the method described in the next section. Once the values $J_t^{(j)}$ are obtained for all contingencies, the classification of contingencies is performed as follows: The contingencies which yield a $J_t^{(j)}$ value greater than 1.0 are analyzed with usual power flow analysis and excluded from the remaining procedure.

Ranking for Voltage Problems

The performance index is defined with

$$J_v = \sum_{k=1}^{N} W_k\left(\frac{V_k - V_{k,av}}{V_{k,st}}\right)^{2n} \tag{7}$$

where:

V_k : the actual voltage magnitude at bus k

$V_{k,av}$ $= \frac{1}{2}\,(V_k^{max} + V_k^{min})$

$V_{k,st}$ $= \frac{1}{2}\,(V_k^{max} - V_k^{min})$

V_k^{max}, V_k^{min} : the maximum and minimum allowable voltage magnitude at bus k

W_k : a weighting factor for bus k

n : an arbitrarily selected integer.

The performance index assumes a small value when all the voltage magnitudes are within the specified voltage limits and assumes a large value when one or more voltages are outside the allowable range. Thus, it provides a measure of voltage "normality" or security in the system. The ranking method involves the computation of the change of the performance index under each contingency, and subsequent ranking of contingencies based on the change. Specifically, for every contingency j described with the parameter p_j, the first order approximation of the performance index change, ΔJ, is computed as follows

$$\Delta J_v = \frac{dJ_v}{dp_j}\,\Delta p_j \tag{8}$$

It is important to note that in all three PI ranking algorithms, the derivative of the performance index is required. This computation is described in the next section. A number of modeling issues are also addressed within the computation of the derivative of the performance index.

Ranking of Class B Contingencies

Ranking of Class B contingencies is achieved with an iterative subnetwork solution based on the fast decoupled power flow. At each iteration, the subnetwork includes only those buses for which the power mismatch is above a prespecified threshold value. This method is a variation of the zero mismatch approach described in Reference [18]. It allows faster solution of contingencies with balanced accuracy.

The overall contingency ranking algorithm is illustrated in Figure 2.

COMPUTATION OF PERFORMANCE INDEX GRADIENT

This section addresses the computation of the gradient of a general performance index. An AC network model is used and the effects of regulators as well as voltage dependent loads are explicitly modeled.

The problem is posed as follows. Given a performance index J, which in general is a function of the system state, x, and contingency parameters, p, compute the derivative $\frac{dJ}{dp}$. Note that the state x and parameters p are interdependent through the power flow equations. To account for the effects of regulating transformers, the taps of regulating transformers t are introduced as state variables. The regulated bus voltages impose additional constraints. Thus the power flow equations may be written as

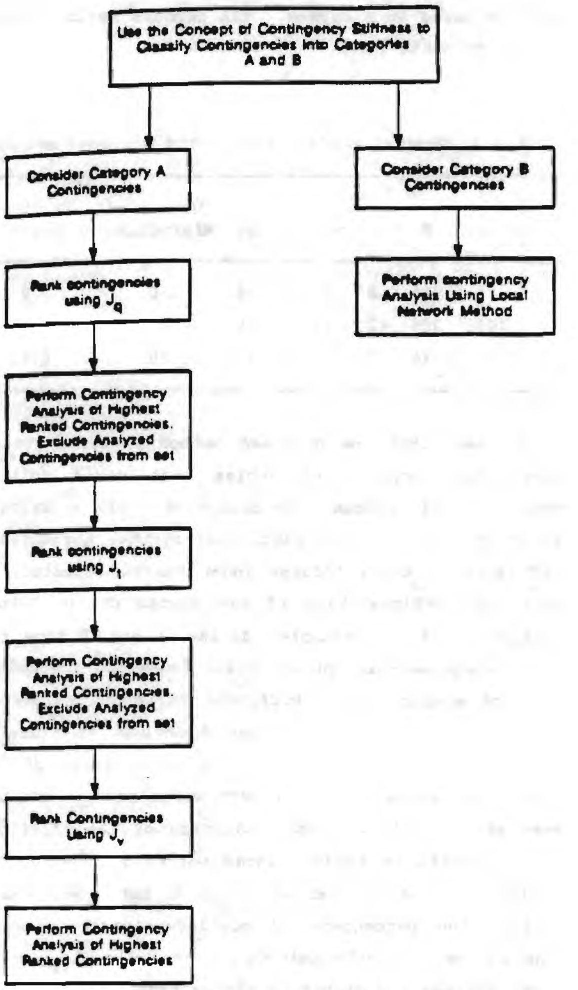

Figure 2 Proposed contingency selection/analysis method

$$g(x,t,p) = 0 \tag{9}$$

$$x_j - c_j = 0 \tag{10}$$

where:

x : the state vector of bus voltage phase and magnitude

t : the vector of tap settings of regulating transformers

p : the vector of contingency parameters; for circuit outage, it is the circuit series admittances; for generating unit outage, it is the unit's power output

g : the system's real and reactive power equations

j : the index of regulating transformers

x_j : the voltage magnitudes at regulated buses

c_j : the specified voltage magnitudes at regulated buses.

We introduce the extended state vector x' defined as

$$x' = \begin{bmatrix} x \\ t \end{bmatrix} \tag{11}$$

and the extended power flow equation

$$g'(x',p) = \begin{bmatrix} g(x,t,p) \\ x_j - c_j \end{bmatrix} = 0 \tag{12}$$

The derivative of the performance index is computed with the equation

$$\frac{dJ}{dp} = \frac{\partial J}{\partial p} - \hat{x}^T \frac{\partial g'(x',p)}{\partial p} \tag{13}$$

where $\hat{x}^T$ is the costate of the system, defined with

$$\hat{x}^T = \left(\frac{\partial J}{\partial x'}\right)^T \left(\frac{\partial g'(x',p)}{\partial x'}\right)^{-1} \tag{14}$$

In the equations above, the term $\frac{\partial g'}{\partial x'}$ is the Jacobian matrix of the extended power flow equations (12). It consists of the usual Jacobian matrix and some added entries resulting from the introduction of the tap variables, t, in the state vector and the regulation equations (10). The costate vector $\hat{x}$ is independent of the contingency parameter vector p and needs to be computed only once. Thus, the computation of the derivative (13) requires the following steps:

(1) Formation and factorization of the modified Jacobian matrix.

(2) Computation of the vector $\frac{\partial J}{\partial x'}$.

(3) Computation of the costate vector $\hat{x}$ with one forward and back substitution from Eq. (14).

(4) Computation of the scalar $\frac{\partial J}{\partial p}$.

(5) Computation of the sparse vector $\frac{\partial g'(x',p)}{\partial p}$.

(6) Substitute the results of steps (3), (4), and (5) into Eq. (13).

Steps (1), (2), and (3) need to be performed only once. Subsequently, for each contingency, steps (4), (5), and (6) are performed to obtain the derivative $\frac{dJ}{dp}$. These computations are comparable to one iteration of the Newton-Raphson power flow.

The algorithm of computing the derivative is general and any performance index can be accommodated. There is no constraint on selecting the exponent value n. Higher exponents are used in Eqs. (3), (5), and (7) to reduce masking effects.

Modeling of Voltage Dependent Loads

A voltage sensitive electric load is represented as a load consisting of two components: (1) a constant power load (independent of bus voltage) and (2) a constant impedance load. The decomposition of the load into these two components is done in such a way as to match the incremental dependency of the bus load on the voltage:

$$\frac{dP_{ti}}{dV_i} = 2g_i V_i \tag{15}$$

$$\frac{dQ_{ti}}{dV_i} = 2b_i V_i \tag{16}$$

where:

P_{ti}, Q_{ti} : the total real and reactive power load of bus i

g_i, b_i : the conductance and susceptance of the constant admittance load at bus i

V_i : the voltage of bus i at base case.

TEST RESULTS

The proposed contingency ranking method has been tested on the following systems.

(1) A 24 bus system (Reliability Test System, RTS)

(2) A 308 bus system of a Northeastern utility

(3) A 1304 bus representation of Georgia Power Company.

The contingency ranking algorithm of Figure 2 has been applied to the three systems. For the purpose of monitoring the performance of the method, the following are defined:

N : total number of contingencies

n_o : Class B contingencies

$N-n_o$: Class A contingencies

n_1 : Class A contingencies ranked as severe using performance index J_q

n_2 : Class A contingencies ranked as severe using performance index J_t

n_3 : Class A contingencies ranked as severe using performance index J_v.

A ranking and analysis process is performed. Ranking consists of computing the first order approximation of the performance index. Analysis consists of solving a full AC power flow for contingencies starting from the top of the ranking list. The analysis will stop if k consecutive success cases are encountered. Here a success case means that there is no voltage constraint violations in the post contingency state, while a failure case means at least one voltage constraint is violated.

Table 1 lists the value N, n_o, n_1, n_2, and n_3 for the three systems. The number of misrankings is determined as follows: For all the contingencies which have not been analyzed with the proposed contingency ranking algorithm, the AC power flow is solved. The voltage constraints are checked for each post contingency state. The total number of failure cases is considered the total number of misrankings. The number of misrankings is listed in Table 1. The capture ratio is defined as the ratio of the number of the identified failure cases to the number of total failure cases in a system. The capture ratio is also listed in Table 1.

Table 1 General performance of the proposed method

System	N	n_o	n_1	n_2	n_3	Number of Misrankings	Capture Ratio
24	36	5	2	4	8	0	1.0
308	266	42	2	2	18	3	0.91
1304	1546	77	4	2	51	10	0.83

It is seen that the proposed method is effective in identifying most contingencies that cause voltage constraint violations. In case that a higher capture ratio is desired for a particular system, the threshold value of the stiffness index can be adjusted, so that the nonlinearities of the system can be better analyzed. As an example, Tables 2 and 3 show the correlation between the stiffness index and the reduction of misrankings. Different threshold values of the stiffness index, S_{TH}, have been used to classify the class B contingencies. The first value of S_{TH} listed in Tables 2 and 3 is the value used to get the results in Table 1. The reduction of the threshold value results in obvious improvements in the capture ratio. As seen in Tables 2 and 3, for a particular system, the percentage of contingencies in class B depends on the selected threshold value S_{TH}. For large systems, a capture ratio as high as 0.95 can be

Table 2 Correlation between stiffness index and reduction of misrankings (308 bus system)

S_{TH}	Percentage of Contingencies in Class B	Number of Misrankings	Capture Ratio
0.09	15.78	3	0.91
0.08	21.42	2	0.94
0.07	26.69	1	0.97

Table 3 Correlation Between Stiffness Index and Reduction of Misrankings (1304 bus system)

S_{TH}	Percentage of Contingencies in Class B	Number of Misrankings	Capture Ratio
0.09	4.98	10	0.83
0.07	9.31	6	0.90
0.05	19.56	3	0.95

achieved with about 20% of the contingencies being included in class B. This means that by using the stiffness index to classify contingencies, it is possible to make optimal use of both performance index ranking methods and subnetwork solution methods.

Another advantage of the proposed method is that there is no constraint on choosing the exponent value, n, in a performance index. Tests show that for some systems, using a higher exponent value leads to better ranking results. An exponent value n = 2 provides overall good results. The results of Tables 1, 2, and 3 have been obtained with n = 2.

The efficiency of the proposed method is very good. Execution times have been measured on a VAXstation 3200 computer system.

Table 4 shows the measured computation time of the proposed contingency ranking method. Note that the computation of the first order change of a performance index for all contingencies in a system is extremely fast, and the time requirement is almost proportional to the size of the system. The average solution time per contingency in class B is somewhat high because this part of the code has not been optimized. In other words, there is room for substantial improvements in the subnetwork solution method.

Table 4 Computation time in seconds on a VAXstation 3200 computer

	308 bus system	1304 bus system
Base Case Solution by Fast Decoupled Method	4.0	12.0
Computation of First Order Change of PI for all Contingencies	0.8	3.5
Average Solution Time per Contingency in Class B	2.2	6.8

CONCLUSIONS

A hybrid contingency ranking method is proposed for voltage security assessment. The method consists of classification of contingencies and PI based ranking methods. It is capable of accounting for several causes of misrankings i.e. nonlinearities of reactive power equations, and discontinuities resulting from unit reactive power limits and regulator tap limits.

The concept of contingency stiffness is introduced to classify contingencies. Tests show a high correlation between the degree of nonlinearity in a system and the contingency stiffness. Contingency stiffness is used as a criterion to determine a priori which contingencies can be well predicted with a performance index ranking method. On the other hand, the effects of discontinuities are predicted with a PI method. Discontinuities that have been addressed are those resulting from reactive power generation limits and regulating transformer tap limits.

The significance of the proposed contingency classification is that the validity of PI based ranking methods for voltage problems can be predetermined. In this sense, it is possible to take full advantage of the fast PI based ranking algorithm for a subset of contingencies which turn out to be the majority. The remaining contingencies are ranked with a screening method based on subnetwork solutions.

The proposed contingency ranking algorithm in this paper is efficient and the implementation is straightforward. The innovations of the method include the explicit modeling of voltage regulators and voltage sensitive load. Preliminary tests demonstrate the superiority of the method in identifying severe contingencies with high capture ratio.

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APPENDIX F: Performance Evaluation of a Subnetwork Solution for Contingency Selection

C. Cheng and A. P. Sakis Meliopoulos, "Performance Evaluation of a Subnetwork Solution for Contingency Selection," Proceeding of the 22nd Annual North American Power Symposium, pp. 348-362, Auburn, Alabama, October 15-16, 1990.

Performance Evaluation of a Subnetwork Solution for Contingency Selection

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Introduction

To ensure secure operation of a power system, the effects of numerous contingencies need to be assessed in real time. Conceptually, the solution of an AC power flow for every contingency is required. This brute force approach is unpractical because of excessive computational times. Many years ago, another practical two-step approach has been introduced: In step one, contingencies which may have an adverse effect on system security (critical contingencies) are identified with a certain method (contingency selection). In step two, only the critical contingencies are analyzed with the AC power flow. One of the proposed methods for contingency selection is based on the subnetwork solutions. Several variations of subnetwork solutions have been proposed[1,2,4,5,7]. The first attempt was based on concentric relaxation and the Gauss-Seidel algorithm[1,2]. Later, as new developments in sparsity techniques became available[3], a bounding method has been proposed based on a subnetwork solution for the state variables with the fast forward and fast back substitutions[4,5]. Most recently, a generalized subnetwork solution method has been proposed under the name zero mismatch approach[7, 8].

The development of the sparsity-oriented subnetwork solution methods[4,5,7,8] relies on two important facts: first, most contingencies cause localized disturbances, i.e. the power imbalance resulting from a contingency is absorbed mainly by buses close to it. The power mismatch at buses far away from the outage is very small, and the state variables at these buses remain almost unchanged. Therefore, it is reasonable to assume that small power mismatches below a threshold will have a negligible effect on the state variables. Second, the affected local area does not need a separate model for solution. With the aid of sparse vector methods, the state variables of the disturbed area can be solved using a subset of the LU factors of the system.

The computational savings of the sparsity-oriented subnetwork solution is achieved by performing only the necessary operations in the substitution. This substitution scheme is known as the fast forward and fast back (FF/FB) substitution[3]. In a subnetwork solution, before the FF/FB is performed, a local network must be predicted. It includes all the buses with large power mismatches and expected substantial state variable changes. This task results in a substitution path which indicates the order of the forward and back substitution. The path must be updated for every modification of the local network. If the mismatch vector and the solution vector are not sufficiently sparse, then the prediction of the local network, the computation of the path and the substitution itself may require more time than the conventional AC power flow solution with the full forward and back substitutions.

In a practical power system, there are contingencies which cause large disturbances. During the first few iterations of the power flow solution, the power mismatches at many buses are not negligible, and many state variables change significantly from their precontingency values. This situation has been observed among contingency cases resulting in high degree of nonlinearities[6]. As an example, Table 1 shows the effects of selected contingencies on a 1304 bus system. The first two columns show the percentage of buses whose mismatches are above a threshold in the first iteration of the Fast Decoupled Power Flow. Similarly, the last two columns show the percentage of buses with a change of state variables above a threshold. It should be apparent from this table that the subnetwork solution is advantageous for some contingencies but not for others. This paper suggests that the efficiency of the subnetwork solution as compared to the full solution depends on the contingency type and the system size.

The purpose of this paper is to examine the computational efficiency of subnetwork solutions. For this purpose, a statistical approach is used to evaluate the performance of the subnetwork solution method on several systems. For

Table 1 Effects of Selected Contingencies (1304 Bus System)

Contingency No.	No. of cir. outage	% of buses with P-mismatch > 2.0 MW	% of buses with Q-mismatch > 2.0 MVar	% of buses with δ change > 4.0°	% of buses with V change > 0.02 pu
1	1	0.2	10.0	7.2	6.8
2	1	0.2	7.8	99.8	1.9
3	1	0.2	3.3	16.3	1.9
4	1	0.2	2.4	0.2	1.9
5	1	0.2	2.2	2.1	2.0

convenience, the sparsity-oriented subnetwork solution with FF/FB will be referred to as the subnetwork solution, while the conventional power flow solution with the full forward and back substitution will be referred to as the direct solution. Next, a brief description of the evaluation procedure is presented, followed with the evaluation results obtained from two large power systems. Finally, possible application of the performance evaluation results is briefly discussed.

Performance Evaluation Procedures

The performance of the subnetwork solution is evaluated with a statistical approach. Specifically, a comparison of the computational efficiency is made between the subnetwork solution and the direct solution. Both methods are applied to solve the equation **Ax = b**, where **A** is any power flow matrix, **b** is the power mismatch vector and **x** is the solution vector. The evaluation procedure is carried out with repeated trials. On each trial, first, a contingency is selected randomly. A local network is defined with n tiers of buses around the buses of the removed circuit. The mismatch vector is defined as follows: if a bus is not in the local network, its mismatch value is zero. Otherwise, a nonzero value, such as unity, is assigned as the mismatch at this bus. Next, both the subnetwork solution and the direct solution are performed and the computation times are recorded. When a large number of trials are performed, a histogram of execution times is generated. The flow chart of the above procedure is illustrated in Figure 1.

Note that two more steps are required by the subnetwork solution: Defining the n-tier subsystem and forming the path. These steps are not needed in the direct solution.

Performance evaluation Results

The performance of the subnetwork solution has been measured on the following power systems:

(1) A 308 bus system of a northeastern utility,

(2) A 1304 bus representation of Georgia Power Company.

Without loss of generality, the B′ matrix of each system is utilized. The following three substitution schemes are examined:

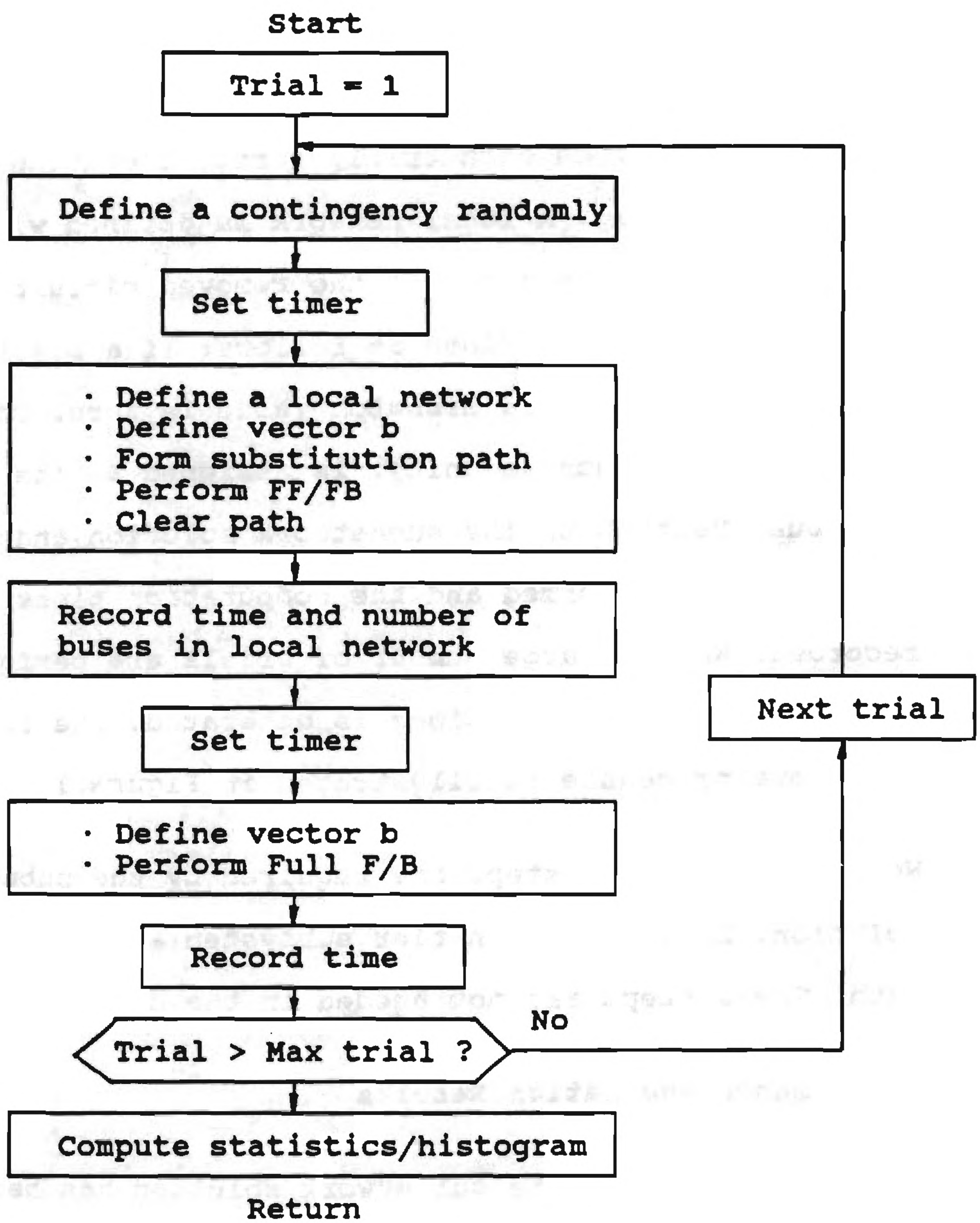

Figure 1 Flow Chart of the Performance Evaluation Procedure

(1) FF on vector **b** defined by n tiers of buses, FB for the solution variables of n+2 tiers of buses,

(2) FF on vector **b** defined by n tiers of buses, full back substitution,

(3) Full forward and back substitutions.

The local network is selected with n = 0, 1, 2, 3 and 4, respectively. Here zero tier means that the local network comprises only the buses previously connected to the outage. The outcome of each trial includes the computation time and the number of buses in the local network. Following the analysis, histograms of the recorded computational times are formed. The average computational times, μ_1, μ_2, μ_3 (for the three substitution schemes, defined above), the average percentage of nonzeros in the mismatch vector, p_F, and in the solution vector, p_B, are also computed.

The histogram for the 304 bus system is shown in Figure 2. In Figure 2 (a), (b) and (c), the histogram of the three substitution schemes are displayed, while in Figure 2 (d) and (e), the histogram of the scheme FF/Full back is not included. Similarly, the histogram of the 1304 bus system is shown in Figure 3. The average computational times in milliseconds and the average percentages are listed in Table 2 and Table 3, respectively.

The following observations are relevant to the performance of the subnetwork solution:

1. When the mismatch vector is very sparse and the solution vector is also fairly sparse (n=0 or 1 cases), the histogram indicates that the subnetwork solution is comfortably

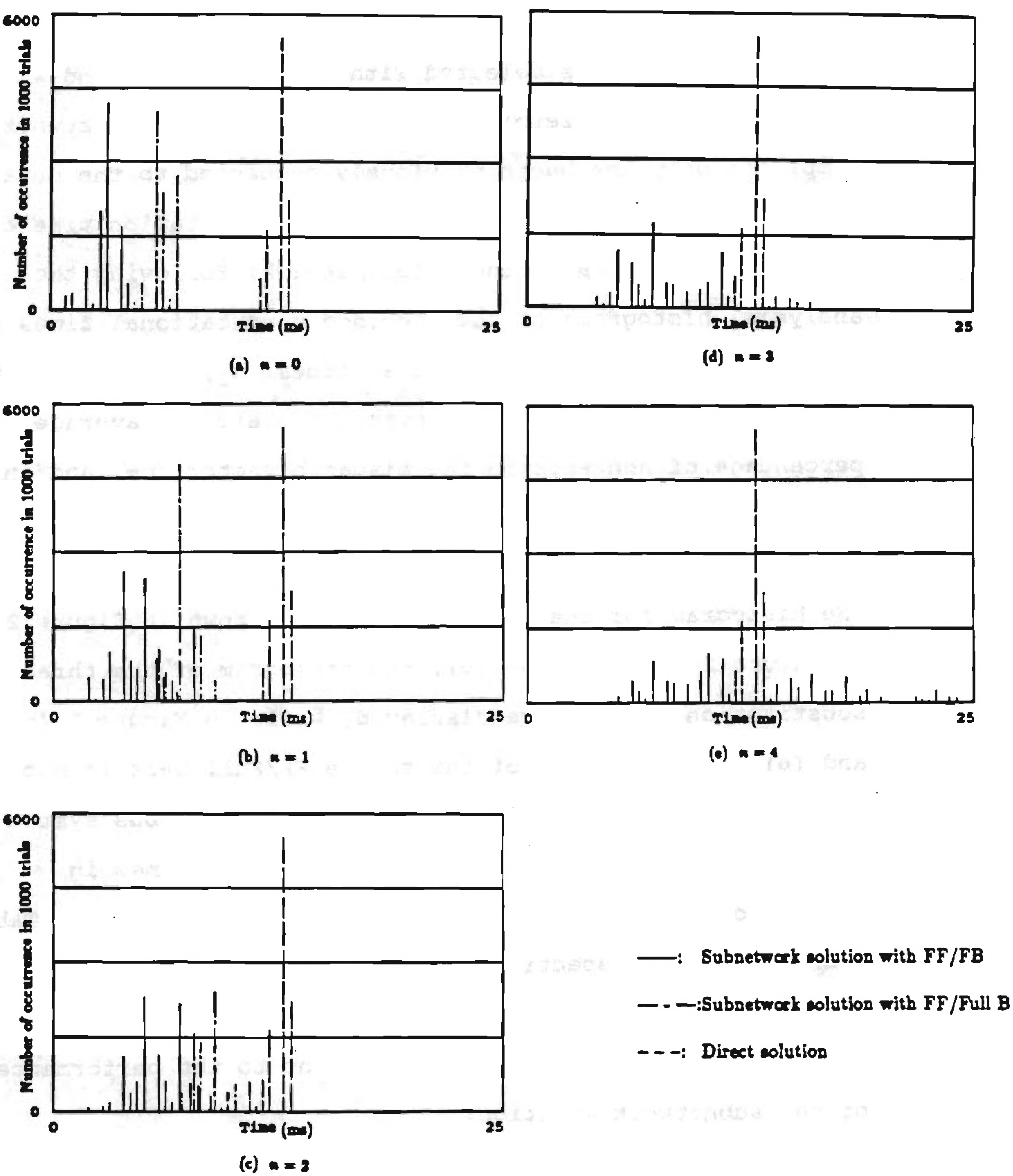

Figure 2 Histogram of computation time (308 bus system)

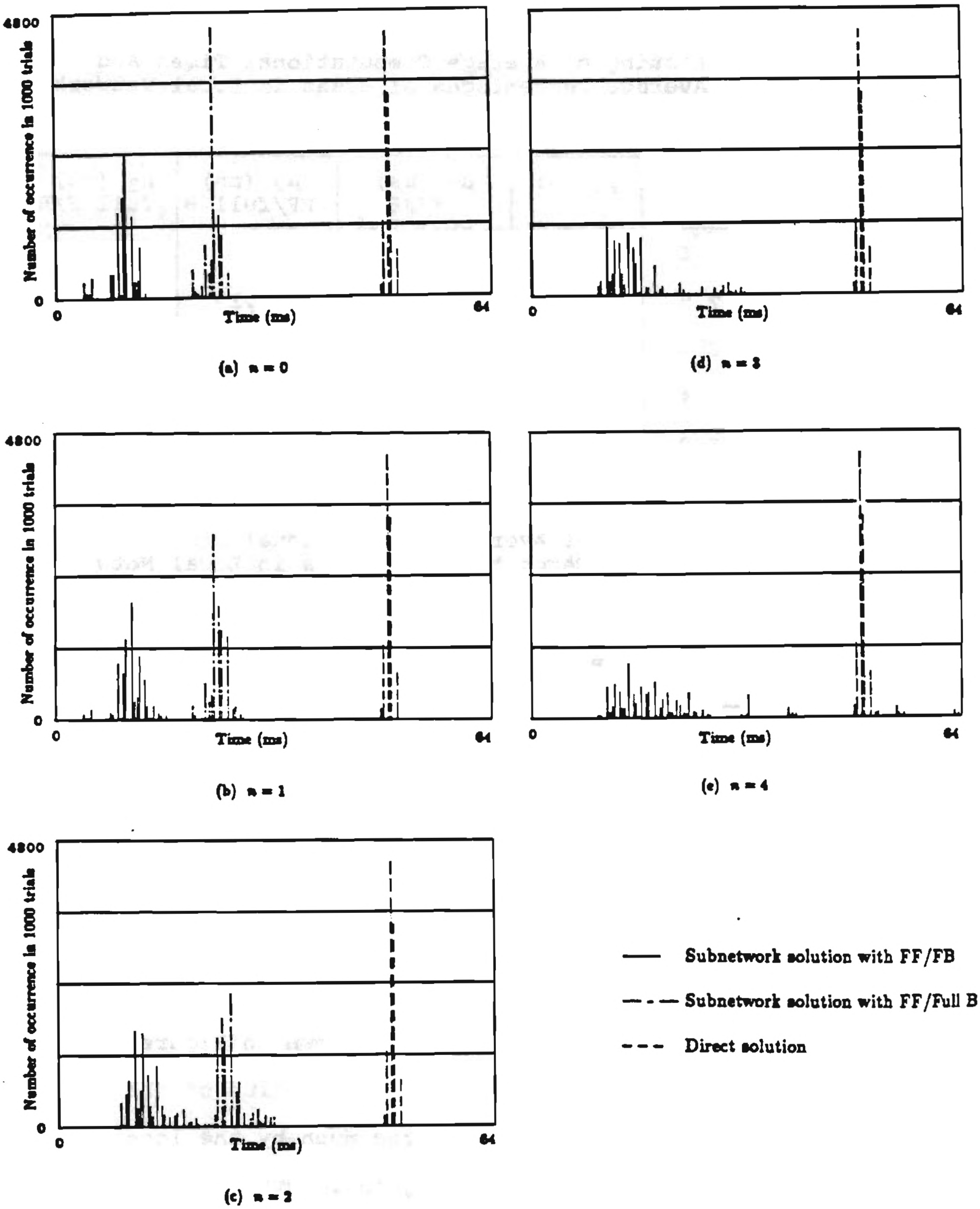

Figure 3 Histogram of computation time (1304 bus system)

Table 2 Listing of Average Computational Times and Average Percentages of Buses in Local Network (308 bus system)

No.of Tiers	p_F (%)	p_B (%)	μ_1 (ms) FF/FB	μ_2 (ms) FF/full B	μ_3 (ms) full F/B
0	0.6	6.3	2.9	6.2	12.5
1	2.8	11.4	4.3	7.1	12.5
2	6.4	18.4	6.3	8.4	12.5
3	11.4	27.5	8.9	10.6	12.5
4	18.0	38.5	12.3	14.0	12.5

Table 3 Listing of Average Computational Times and Average Percentages of Buses in Local Network (1304 bus system)

No.of Tiers	p_F (%)	p_B (%)	μ_1 (ms) FF/FB	μ_2 (ms) FF/full B	μ_3 (ms) full F/B
0	0.2	1.0	7.9	22.9	48.5
1	0.4	1.8	10.2	23.6	48.5
2	0.9	3.3	12.1	25.0	48.5
3	1.9	5.7	15.1	27.5	48.5
4	3.3	9.4	22.1	32.4	48.5

superior to the direct solution, as shown in Figure 2(a)(b) and 3(a)(b). Moreover, the computational time of the subnetwork solution is not affected much by the locations of the outage. In other words, the path length does not vary much with the locations of the nonzero elements in the vector. This result coincides with the one obtained in [3], where singleton vectors are used for testing. On the contrary, if the mismatch vector and the solution vector are less sparse (n=4 case), then the computational times are substantially affected by the location of outages, as seen in Figure 2(e) and 3(e).

2. The computational advantage of the subnetwork solution is dependent upon the size of the local network. As an example, for the 308 bus system, if the local network is defined with up to two tiers, i.e. on the average, 6.4% nonzero elements in the mismatch vector and 18.4% nonzero elements in the solution vector, then the subnetwork solution is definitely superior to the direct solution. On the other hand, there is a large chance for the subnetwork solution to be slower than the direct solution, if the local network is defined with three tiers. In this case, on the average more than 15% nonzero elements are in the mismatch vector, and 35% in the solution vector. Note that above timing figures depend a on programmer's ingenuity. Nevertheless, given a power system, a threshold percentage exists, above which the advantage of the subnetwork solution over the direct solution vanishes or reverses.

3. The advantage of the subnetwork solution is more significant for larger systems (i.e. 1304 bus system) than smaller systems (Figure 3(d)(e)). However, depending on the structure of the system network, lower percentage of nonzero elements may be required to ensure the superiority of the

subnetwork solution. This can be seen in Figure 4, where the subnetwork is defined with n=6. The advantage of the subnetwork solution is reduced significantly even if the percentage of nonzero elements in the mismatch vector is lower than 10%.

4. The computational efficiency of the subnetwork solution with FF/Full back is between the one with FF/FB and the direct solution. If the mismatch vector is very sparse (n=0 or 1 cases), this scheme is more efficient than the subnetwork solution with FF/FB on less sparse vectors (n=3 or 4 cases). This can be seen by comparing, for example, Figure 3(a) or (b) with Figure 3(e). In case that the nonzeros in the mismatch vector is easy to determine, but the change in the state variables is hard to predict, the subnetwork solution with FF/Full back may be a practical alternative for the subnetwork solution with FF/FB.

Possible Applications of the Performance Evaluation

The results of the performance evaluation can be used to control the algorithm of the approximate power flow solution in an overall contingency selection method. For example, in case of a single transmission line outage, there will be immediate power mismatches at buses previously connected to the outaged line. This power mismatch may disturb the state variables at many buses, as previously shown in Table 1. Based on the previous results, it is

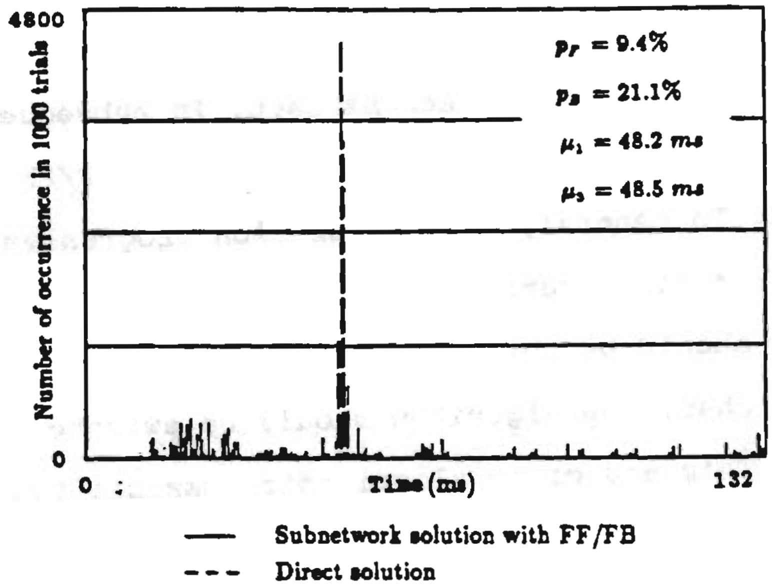

Figure 4 Histogram of computation time
(1304 bus system, $n = 6$)

expedient that for the first iteration of the Fast Decoupled Power Flow, a FF/Full back be used. In subsequent iterations, the subnetwork solution with FF/FB should be applied. In general, as the solution progresses, the actual percentage of nonzero elements in the mismatch and solution vectors should be checked. If a prespecified percentage has been reached, the algorithm should be switched to the direct method. This and other algorithmic possibilities are being investigated.

Conclusions

The subnetwork solution method provides a promising approach for contingency selection. In order to take full advantage of the method, the performance of the subnetwork solution is evaluated in detail with a statistical method. The results show that the efficiency of the subnetwork solution depends on the location of the contingency and the size of the disturbed area. For certain contingencies, the advantage of the subnetwork method over the direct method may be lost. In general the subnetwork method performs better for larger systems. The results of the performance evaluation can be used to design adaptive algorithms for contingency selection.

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APPENDIX G: Modeling and Optimization Issues In Expansion Plan Evaluation Methods

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MODELING AND OPTIMIZATION ISSUES IN EXPANSION PLAN EVALUATION METHODS

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MODELING AND OPTIMIZATION ISSUES IN EXPANSION PLAN EVALUATION METHODS

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ABSTRACT

Recent trends in the electric power industry have affected operating and optimization practices. These changes must be reflected in planning studies. In this context, modeling and optimization issues arise. This paper discusses modeling and optimization issues as dictated by recent trends and developments in the electric power industry. It proposes a new electric load model and a new power flow/optimization formulation. The need for composite power system simulation methods is discussed and recent results using the probabilistic simulation method are presented. Sensitivity analysis embedded in a composite power system simulation method is proposed for comparative evaluation of expansion plans.

1. INTRODUCTION

The electric power industry is undergoing unprecedented changes. Technology advances, the PURPA legislation, and an increasingly competitive market are transforming the traditional ways of operating and planning an electric power system. From the planning point of view, the following major changes have occurred:

1. The uncertainty associated with future parameters has increased.
2. The electric load is not an exogenous process anymore but rather a process which depends on power system operating cost and practices.
3. Expansion plans seek least cost in which independent power producers, qualified facilities, load management options, etc., compete in a complex regulated environment.
4. The electric power utility is changing its approach to marketing following a more aggressive approach in which value based marketing is central.

These trends have made the traditional approach to power system planning obsolete. A fresh approach is needed to cope with the problem of evaluating the large number of options facing the planner today. What makes the task difficult is that the large number of options have quite different characteristics and impact on power system economics and operating security. A complete evaluation of expansion options should account for:

1. The impact of an expansion option on several power system attributes such as cost, reliability, security, etc.
2. Sensitivity analysis of system attributes with respect to expansion options.
3. Trade-off analysis.
4. Effects of policies and practices (for example, possible marketing on reliability) on power system attributes.
5. Effects of improved operational optimization.
6. Quantification of reliability and determination of a fair way to share cost of reliability.
7. Quantification of system losses and fair allocation of cost of losses among interconnected systems.

The central issue in the above stated needs is the composite power system simulation in view of expansion option in a system with increased controllable elements (FACTS elements as defined by EPRI). This paper describes some recent developments towards this goal and outlines the scope and objectives of the research. Specifically, methodologies are being developed which focus on technico-economical evaluations of the effects of expansion options on power system operation, security, and transfer capability. Two specific issues are addressed: (1) modeling and (2) optimization. In terms of modeling, a novel electric load model is proposed recognizing load uncertainty, voltage dependence of the load, and capable of modeling wheeling schedules, IPPs, cogeneration, and customers responding to specific rate structures. In terms of optimization, a new formulation of the power flow is proposed which considers effects of voltage regulators, interchange control, control of IPPs, remedial actions involving operation of FACTS elements, and transfer capability. Within this framework a sensitivity analysis is embedded which computes sensitivities of operating costs, security measures, and transfer capability with respect to specific expansion options. The sensitivity analysis forms the basis for incremental cost/benefit analysis. The proposed modeling and optimization methods are embedded in the composite power system simulation method based on three approaches: (1) Enumerative Approach, (2) Monte Carlo, and (3) Probabilistic Simulation.

This paper presents the electric load modeling, the new power flow/optimization formulation, and the composite power simulation methods. Typical results are discussed.

2. ELECTRIC LOAD MODELING

Traditionally, the electric load has been thought of being an exogenous process to the electric power system. Planners will design the system to satisfy the electric load with a certain degree of reliability. Load management programs have change this assumption many years ago. Recent trends will further change the way planners look at the electric load altogether. These trends are: (1) Power Wheeling: Now there are customers which represent positive electric load at one bus and negative electric load at another bus. (2) Cogeneration: The electric load of a cogenerator responding to a specific rate structure may depend on the operating cost of the electric power system. (3) Large customers with non-utility generation and storage capabilities. The electric load of these customers and its time variation depends on electric utility production costs. (4) Small independent producers responding to spot prices and others. The effects of these changes on electric load is twofold: First, the uncertainty associated with the electric load will increase. Second, the electric load is not an exogenous process anymore but a process which responds to the operating cost of the power system. In this environment, a fresh approach in modeling the electric load is needed. The problem of modeling an electric load and its uncertainty has been under investigation for many, many years for reliability analysis and production costing. An electric load model which favorably addresses the needs arising from recent trends has been proposed and implemented in [23].

Figure 1. Electric Load Model Based on Small Number of Independent Processes

The proposed electric load model is illustrated in Figure 1. It is a multiple input/multiple output ARIMA model. Specifically, the bus electric load, represented with the vector S(t), is constructed from a vector of m independent white noise processes $\eta(t)$. When the white noise vector $\eta(t)$ passes through an ARMA model, it generates an m-vector of stationary stochastic processes z(t). Next, the vector z(t) is inverted to provide an m-vector of nonstationary stochastic processes x(t). Finally, the vector x(t) is translated into bus electric loads using the linear system L. The described model is represented with the following equations:

$$\phi_1(B)z(t) = \phi_2(B)\eta(t) \tag{1}$$

$$\phi_3(B)z(t) = x(t) \tag{2}$$

$$P(t) = Lx(t) \tag{3}$$

$$S_i(t) = P_i(t) + j\alpha_i P_i(t) \tag{4}$$

where

$\eta(t)$	is an m-vector of independent white noise processes
$z(t)$	is an m-vector of stationary stochastic processes
$x(t)$	is an m-vector of nonstationary stochastic processes
$P(t)$	is an n-vector of bus real power electric loads
$S_i(t)$	is the complex power of the electric load at bus i
α_i	is a constant for bus i
$\phi_1(\cdot),\phi_2(\cdot),\phi_3(\cdot)$	are vectors of arbitrary polynomials of the argument
B	is the backward operator
L	is an nxm matrix, m << n.

ARIMA models have been extensively used to represent the electric load. It is well known that they are capable of representing the periodicities as well as the nonstationary property of the electric load. The innovation introduced here is the linear model L which translates the low order nonstationary stochastic process vector x(t) into the high order vector P(t) of the bus electric loads. This innovation is justified on the basis that bus electric loads are typically strongly correlated. It is, therefore, reasonable to assume that they are generated as a linear combination of a small number of independent stochastic processes.

The optimal order of the ARIMA model (order of functions, ϕ_1, ϕ_2, and ϕ_3) and the number of independent white noise processes (vector $\eta(t)$) is system dependent.

The Non-Utility System Model. Non-utility systems, such as customer owned generation, power "wheeling" schedules, etc., are represented as electric power injections at specified system buses. These injections are assumed to be stochastic processes. The use of ARIMA models is proposed for this purpose. The advantages of the ARIMA models are (1) they can accurately represent the periodic nature of power "wheeling" schedules, customer owned generation patterns, etc., and (2) they provide a good model to represent the uncertainties associated with customer owned generation, power "wheeling" schedules, etc.

Another advantage of the ARIMA model relates to the possibility that the operation of non-utility systems may be controlled by the utility under certain conditions and constraints. In this case, the customer owned generation patterns, power "wheeling" schedules, etc., may be altered in order to optimize a given operational objective subject to specific constraints. In this case, the ARIMA model parameters can be selected by a proper constrained optimization problem. The resulting ARIMA model will describe the statistics of the non-utility generation and/or the statistic of electric loads responding to specific rate structures. The same approach is applicable to modeling of load management programs.

3. POWER FLOW/OPTIMIZATION MODEL

The power system of the future will rely on FACTS elements to attain an acceptable operating condition. This means that standard power flow algorithms may diverge. A new approach is proposed for power flow analysis which within its algorithm will be able to dispatch FACTS elements as necessary. Utilization of FACTS elements is viewed as remedial actions which are applied by an optimization model within the power flow solution algorithm.

The following formulation is proposed which achieves this goal. Consider an electric power system and an arbitrary state defined with the vector x (the vector x is defined in terms of bus voltage magnitudes and phases). For the assumed state x, let v_i, w_i be the real and reactive power mismatch, respectively, at bus i. Then consider the following optimization problem:

Minimize: $\sum_i |v_i| + |w_i|$

Subject to: Power Balance Equation

Voltage Constraints

Circuit Flow Constraints

Net MW Interchange Constraints

Unit Reactive Power Output Constraints

And Other Pertinent Constraints.

The above problem is efficiently solved with the remedial action methodology described in [22]. The remedial action methodology is based on a linearized model which operates on a computed region of validity of the linearized model. This means that the solution to the above problem may yield nonzero values for the mismatch variables v_i, w_i. A nonzero value of the variables v_i, w_i means that the mismatch at a bus has not been zeroed and, therefore, the algorithm has not converged yet. The computed remedial action is then implemented. These iterations are guaranteed to converge because the remedial action solution guarantees that the solution will move only a small "distance" within the computed region of validity of the linearized model. The process is repeated until the variables v_i, w_i become zero. If a solution cannot be found with all variables $v_i = 0$, $w_i = 0$, then load shedding may be added as a remedial action. The overall algorithm is illustrated in Figure 2. Note that for mild mismatches, the remedial action procedure is not called. Mild mismatches are identified by the impact of the mismatches on the linearized equations of the system around the operating state.

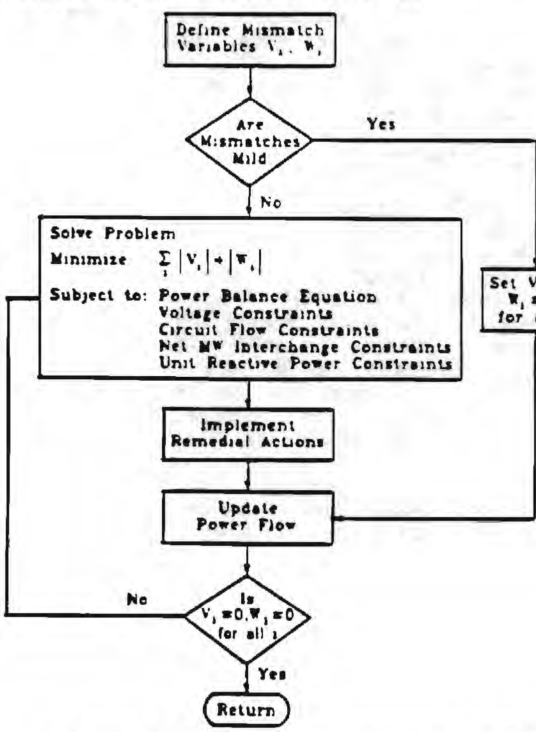

Figure 2. Power Flow/Optimization Model

The proposed approach has several advantages. First, it combines the remedial actions with the power flow solution and, thus, increases the efficiency of the overall model as compared to performing a power flow solution and then a remedial action computation separately. Second, it eliminates the necessity of adjustments during the power flow solutions such as net MW export adjustments, capacitor/reactor switching with local logic, etc. These adjustments may require several power flow iterations. Third, and most important, it will guarantee that severe power mismatches will not result in nonconvergent power flows.

Efficiency-wise, the proposed power flow/optimization algorithm requires overall less execution time than the usual power flow with interchange adjustments, capacitor/reactor switching, etc. In addition, the proposed formulation results in an operating condition which is optimized by taking advantage of the controllability of FACTS elements.

4. COMPOSITE POWER SYSTEM SIMULATION

Power system simulation has been the central method for planning studies and reliability analysis. Traditionally, the generation system is separately simulated from the transmission system. In the past, the design of a power system and its operating practices justified this decomposition. Presently however, and in the future, this decomposition is not justified. Recent trends have resulted in transmission constrained systems which means that there is a substantial impact on reliability, security, and costs due to the interaction between generation and transmission systems. The interaction cannot be ignored anymore, thus the need for composite power system simulation methods.

Efforts to develop composite power system simulation methods date at least 15 years ago. Three distinct approaches are identified in composite power system simulation:

1. The enumerative approach
2. Monte Carlo simulation
3. Probabilistic simulation.

Description and discussion of these approaches follows.

4.1 The Enumerative Approach

The enumerative approach has been extensively used in North America for adequacy evaluation/reliability analysis. The basic idea is to enumerate all possible states of an electric power system, to analyze the state and store the results for subsequent processing. Because of the extreme large number of contingencies, the conceptual and computational problems are serious. As an example, a very large number of multiple generating unit outages must be considered since the probability of these events is substantial (generating unit probability of unavailability is quite high). The problem of a priori determining the severity of multiple unit outages is a challenging problem. An effort toward this goal is the wind-chime method developed by PTI for EPRI. The method is illustrated in Figure 3. The method considers a number of contingency levels. At each level the "binding" contingencies (states) are identified with a multiplicity of contingency ranking methods. The enumerative method is extremely time consuming. It is our strong belief that state enumeration methods are at best ineffective in addressing the present day needs for composite power system simulation (which is the basic tool for reliability analysis). On the other hand, they provide useful and complete information for the enumerated states. As the systems become increasingly complex and stressed to the limits, the need for effective composite system simulation becomes relevant. Aggregate (macroscopic) simulation methods of the composite power system are needed.

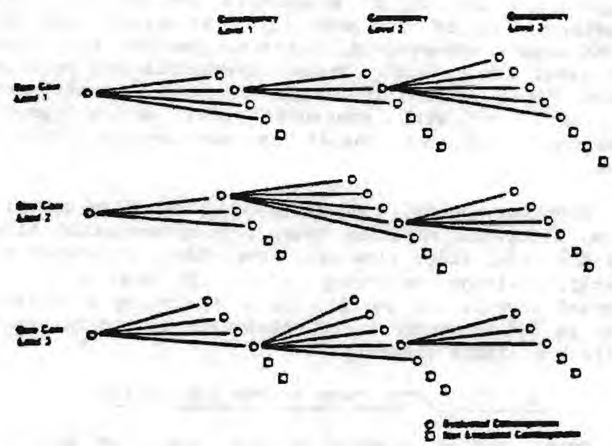

Figure 3. State Enumeration Scheme for Composite Power System Simulation Using the Wind-Chime Method

4.2 Monte Carlo Simulation

Monte Carlo simulation is easily implementable and provides a great tool for validation of other methods. The method is illustrated in Figure 4. It is imperative that the Monte Carlo simulation be based on comprehensive models of electric load, generation, transmission, etc. In this way the results of the Monte Carlo method are directly comparable to the results of the proposed simulation method. Note that in the Monte Carlo method, the number of trials must be large for meaningful results. This requirement hinders the applicability of this method to large scale power systems. The Monte Carlo simulation is typically limited to small or medium size power systems.

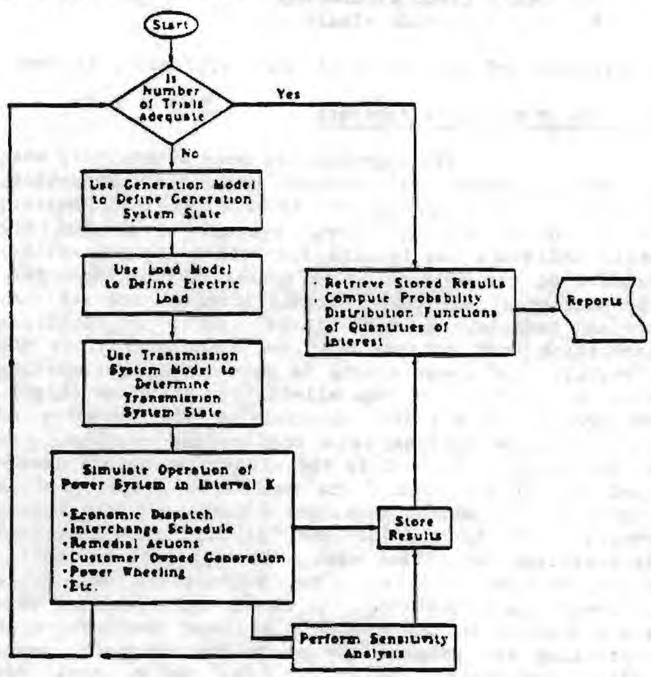

Figure 4. Monte Carlo Simulation

The basic Monte Carlo approach can be applied for each hour in a year in chronological order (sequential approach) or the hours of the study time can be considered at random (random approach). The method requires a stochastic model of the electric load and a stochastic model of component availability. The models can be simple or sophisticated (inclusion of weather effects, econometric parameters, etc.). The simulation of the randomly selected system conditions is done with the use of power system analysis methods, such as load flows, dispatch algorithms, optimization algorithms, and models simulating operating policies. The result of the simulation are probability distribution functions of power system output variables (circuit flows, voltage levels, energy curtailment, etc.). These results are utilized in the computation of appropriate production quantities or reliability indices. The key issues in this approach are: (1) the number of trials must be large enough to adequately capture all possibilities of adverse effects on the system, and (2) the analysis of the effects for a specific trial must be as close to real world as possible and as efficient as possible.

The Monte Carlo approach has several advantages and disadvantages. Consider, for example, the application of this approach for reliability assessment. For meaningful results, it is imperative that a sufficiently large number of problematic system conditions be captured and simulated. However, the majority of the selected trials are problem free. This implies that the Monte Carlo method calls for the expenditure of considerable computing time in order to obtain sufficient confidence in the results. On the other hand, the method allows the analysis of complex systems without forcing the system model to become unrealistic. In addition, it offers a synthesis of the final results and a detailed description of the events that caused the results.

4.3 Probabilistic Simulation

The probabilistic simulation involves the utilization of a probabilistic model for the generation and transmission systems which are coupled with a probabilistic power flow method. The overall procedure is described as follows:

Consider the proposed stochastic load model and the Markov model of generating unit availability. Further, consider the operation of the system during a specified period of time. This period of time may be contiguous or noncontiguous (for example, 1 pm to 3 pm each day for a period of one year). Utilizing the proposed electric load model, the electric load of a bus, during the period of simulation, can be characterized as a random variable with a probability distribution function and correlation to other bus loads. The total electric load can be characterized as a random variable, L, with a probability distribution function, $F_L(\ell)$. Similarly, utilizing the Markov model of generating units, the unit availability is represented as a discrete random variable. Assume that the n units of the system operate at levels $x_1, x_2, \ldots, x_n$, respectively, while the total electric load is ℓ. If unit k is not in operation, then obviously x_k equals 0. Since there is a finite probability that any unit can be forced out, the output of unit j, x_j, is considered to be a random variable with probability of forced outage equal to q_j. We write

$$Pr(A_j = x_j) = 1 - q_j \quad , \quad x_j \neq 0 \tag{5}$$

$$Pr(A_j = 0) = q_j \tag{6}$$

where A_j is a random variable representing the available capacity of unit j, q_j is the probability of unavailability. The above relationships state that the probability that the output generator j is x_j equals

$1 - q_j$, and the probability that the same output is zero equals q_j.

For the condition that has been considered, the apparent load ℓ_a will be

$$\ell_a = \ell - x_1 - x_2 - \ldots - x_n \; . \qquad (7)$$

Since $\ell, x_1, \ldots, x_n$, are not deterministically known, the above equation can be replaced with its equivalent equation in terms of the corresponding random variables

$$L_a = L - \lambda_1 - \lambda_2 - \ldots - \lambda_n \qquad (8)$$

where L is a random variable representing the total electric load and λ_i is a random variable representing the output of unit i. Since the probability distribution functions of the random variables $L, \lambda_1, \ldots, \lambda_n$ are known and since these random variables are independent, the probability distribution function of the random variables L_a is easily computed with a series of convolutions [9].

If we assume that $\ell_a > 0$ (that is, load exceeds generation), then another unit should be brought into operation or one or more of the operating units should increase their output. Assume that unit i is operating at x_i and that it is selected according to a criterion to respond to any increases in the load. We shall refer to this criterion as the dispatch criterion. It is defined as to satisfy operational practices and constraints. The dispatch criterion can be arbitrary but usually is assumed to be the unit incremental cost. Without loss of generality, x_i may be equal to zero. In general, if $\ell_a > 0$, the output of unit i will increase from x_i to $x_i + \Delta x_i$, where Δx_i is a small increment (1-2 MW). We shall refer to this increment as the block Δx_i. The described formulation and direct application of probability theory yields expressions for the expected energy, cost of operation, required fuel, etc., from the Δx_i increase in the output of generator i. The detailed mathematical formulation is given in [23].

Upon completion of the simulation algorithm, the probability that unit i operates at level x_i or the probability density function of power injection at the generation buses as well as the joint probability density of any generating unit pair has been constructed. Thus, the power injections to the electric power network are characterized as random variables with known probability distribution functions and correlations. This is illustrated in Figure 5. It should be emphasized that the probability distribution functions at generation buses cannot be approximated with Gaussian distributions. This basic result can also provide the performance parameters for each generating unit, such as (1) expected time of operation, (2) expected produced energy, (3) expected production cost, etc. [23].

The proposed method is also capable of simulating the operating practices and constraints of a power system. This objective is achieved by appropriate selection of the dispatch criterion mentioned in the description of the method. Specifically, the dispatch criterion is defined as the sum of the actual operating cost of the generating unit plus a nonlinear penalty function of generating unit output defined with m parameters. The definition of the dispatch criterion is illustrated in Figure 6. The proposed method can accommodate an arbitrary dispatch criterion. The parameters of the penalty function are selected with a probabilistic optimization method which is described later. The solution of the optimization problem provides the parameters of the penalty function, such

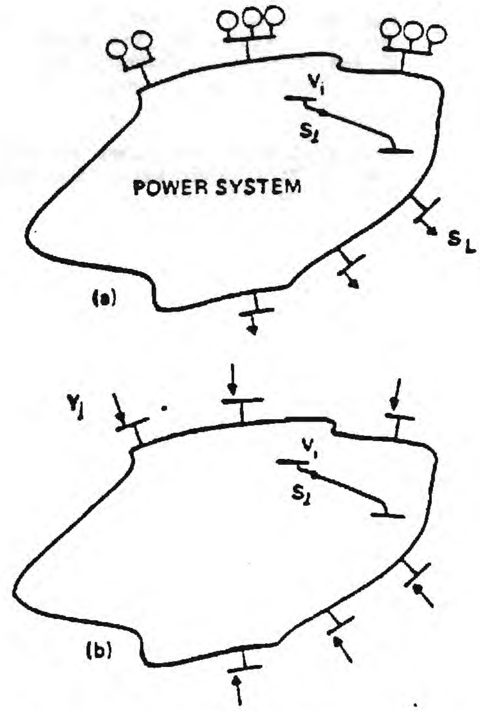

Figure 5. Schematic Representation of an Electric Power System

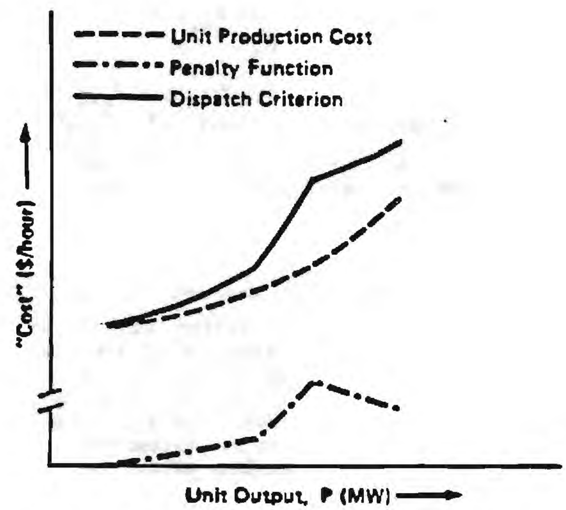

Figure 6. Illustration for the Definition of the Penalty Function and the Dispatch Criterion

as the operational practices and constraints will be satisfied with maximum probability. Note that if the nonlinear penalty function is neglected, then the dispatch criterion equals the actual operating cost of the generating unit. This selection amounts to simulating the economic dispatch only, neglecting other operating practices, such as interchange schedules, etc., and operating constraints, such as hydro energy limitations, transmission limitations, etc.

Once the power injections at the system buses have been probabilistically characterized with the above method, the probabilistic power flow is used to compute the probability density function of system output variables such as circuit loading, bus voltages, and system losses. This transformation can be achieved with any desirable degree of accuracy. The probability density functions of system output variables is translated into reliability indices, total system losses, etc.

The probability distribution functions of output variables is computed by first expressing the output variable as a function of bus power injections:

$$v_{oi} = \bar{v}_{oi} + \sum_j f_{oij}(x_j - \bar{x}_j) \qquad (9)$$

where

v_{oi} is the output variable of interest

$\bar{x}_j$ is the expected value of the power injection at bus j (x may represent real or reactive power)

x_j is the power injection at bus j

$f_{oij}(\cdot)$ is, in general, a nonlinear function of the argument.

For practical calculations, the function $f_{oij}(\cdot)$ is expressed as a piecewise linear function:

$$f_{oij}(x_j - \bar{x}_j) = \{S^k_{oij} y \; ; \; x^{k-1}_j - \bar{x}_j < y < x^k_j - \bar{x}_j \; ; \; k = 1,2,\ldots\} \qquad (10)$$

where

$S^k_{oij} = dv_{oi}/dx_j$ computed at $y = (x^{k-1}_j + x^k_j - 2\bar{x}_j)/2$

References [22] and [23] provide an efficient procedure for computing the sensitivities S^k_{oij} based on the costate (adjoint) equation.

Equation (10) states that the function $f_{oij}(\cdot)$ can be considered linear in an interval $[x^{k-1}_j - \bar{x}_j, x^k_j - \bar{x}_j]$. The interval of validity of the linearized model depends on the parameters of the system in the neighborhood of bus j.

A simple method for selecting the bounds $(x^{k-1}_j - \bar{x}_j)$ and $(x^k_j - \bar{x}_j)$ has been developed and reported in [22]. The method is based ont he observation that the error is approximately an invariant function of the quantities $(P_{inj}/\Sigma Y)$ and $(Q_{inj}/\Sigma Y)$, where P_{inj}, Q_{inj} are real and reactive power injection at a bus and ΣY is the sum of admittance connected to the bus. For a given allowable error, these quantities must not exceed the values ε_1 and ε_2 which depend on the maximum allowable error:

$$\frac{|P_{inj}|}{\Sigma Y} < \varepsilon_1 \qquad (11)$$

$$\frac{|Q_{inj}|}{\Sigma Y} < \varepsilon_2 \qquad (12)$$

Subseqeuntly, Eqs. (11) and (12) are translated into the bounds $(x^{k-1}_j - \bar{x}_j)$ and $(x^k_j - \bar{x}_j)$. It is important to note that the interval of validity of the linearized model is quite wide even for small allowable errors, i.e. 2%. Practically, this means that variations of bus electric loads can be accurately represented with only one segment of the piecewise linear equation (10). More than one segment is needed only at generation buses, or buses with very large loads, where the power injection variations are high. The probability density function of the output variable v_{oi} can be computed from the probabilistic model of the power injections x_j and equations (9) and (10) with direct application of probability theory.

Typical results of this method have been presented in Reference [23]. Figure 7, taken from Reference [23], illustrates the utilization of the method for computing the probability distribution function of circuit 14-16 flow of the IEEE Reliability Test System.

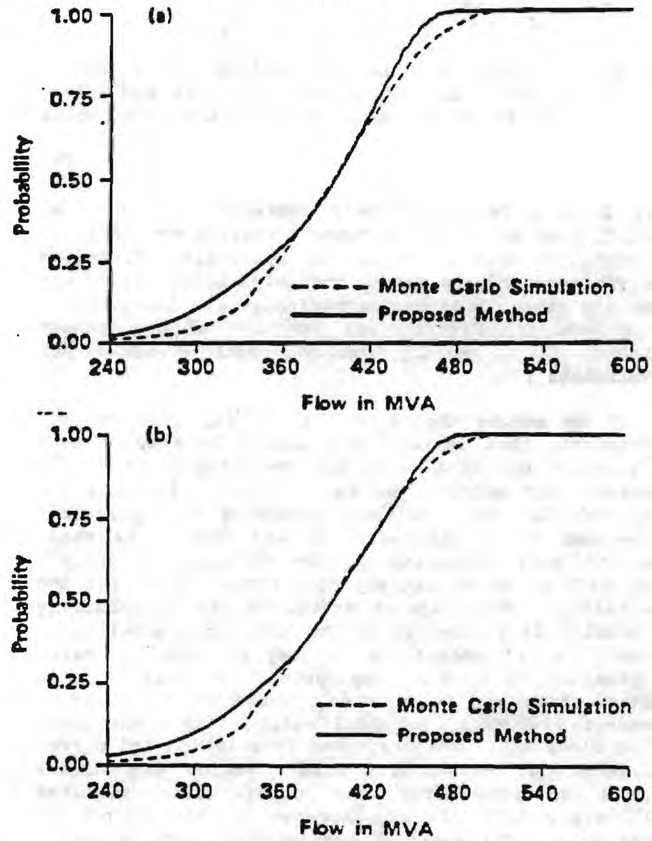

Figure 7. Probability Distribution Functions of Circuit 14-16 Flow
(a) One Segment Load Model
(b) Three Segment Load Model

A difficulty associated with the described probabilistic simulation is the incorporation of transmission system optimization processes or security functions related to transmission limitations. Specifically, probabilistic network models, such as the probabilistic power flow, have not incorporated operating constraints in the formulation.

Presently, an effort has been undertaken to incorporate transmission system operating constraints and practices in the probabilistic power flow. A concise description of this method follows. Recall that the proposed simulation method has been described in such a way that the effects of transmission system constraints on the operation of the generation system are simulated with a set of parameters which define the dispatch criterion. This approach enables the decomposition of the generation system and transmission system simulation methods. The parameters of the dispatch criterion are determined with an optimization problem which optimizes the operation of the system subject to transmission and other operating constraints. The optimization problem is postulated as follows: Determine the required adjustments to the generation bus power injections, such that the probability that an operating constraint violation is minimized. The operating constraints may be: (1) circuit loading constraints, (2) bus voltage allowable range, (3) net power interchange constraints, etc. The mathematical formulation of the probabilistic optimization problem is presented next.

The formulation of the probabilistic optimization is based on modeling the probabilistic operating constraints with deterministic constraints. As an example, consider the power flow T_k of circuit k:

$$T_k = g(x_j \; ; \; j = 1,2,\ldots) \quad (13)$$

where

x_j is the power injection at bus j.

Let $\overline{T}_k$ be the rating of circuit k. Then the probability of overload is defined as:

$$Pr[T_k - \overline{T}_k > 0] = \text{probability that circuit k is overloaded}$$

Recall that the probability density function of the output variable T_k depends on the probability density function of the power injection variables x_j. The latter probability density function depends on the parameters of the penalty function for the generating units and on load shedding if load shedding is allowable. For a mathematical formulation of the problem, define the following control variables:

u_{Gj} is the vector of parameters of the penalty function of generating unit j

u_{Li} is a control parameter expressing load shedding at bus i as follows: the electric load P_{Li}, which is greater than u_{Li}, is shed.

Other control variables may be added if desirable. Then

$$Pr[T_k - \overline{T}_k > 0] = h_k(u_{Gj}, u_{Li} \; ; \; j=1,2,\ldots \; , \; i=1,2,\ldots) \quad (14)$$

The above modeling methodology is repeated to other operating constraints, such as interchange constraints, hydroenergy constraints, bus voltage constraints, etc.

Subsequently, an optimization problem is defined which minimizes the deviation from economic operation subject to constraints that the probability that an operating constraint is violated is less than a prespecified value:

Minimize $\qquad \sum_j |u_{Gj}|$

Subject to

$$Pr[T_k - \overline{T}_k > 0] = h_k(u_{Gj}, u_{Li} \; ; j=1,2,\ldots, i=1,2,\ldots) < P_k$$

$$Pr[E_{hi} - E^{a}_{hi} > 0] = h_i(u_{Gj}, u_{Li} \; ; j=1,2,\ldots, i=1,2,\ldots) = 0$$

Etc.

The defined probabilistic optimization problem can be solved with linear programming techniques. For this purpose, probabilistic constraints are linearized with respect to the parameters u_{Gj}, u_{Li}. This method is in its infancy and no results are available.

5. SENSITIVITY ANALYSIS

Sensitivity analysis is very important for the comparison of alternative options of system expansion of control devices (FACTS). The sensitivity analysis is performed on a selected set of power system attributes over a representative range of power system operating conditions. A representative set of power system attributes is (1) operating cost, (2) security measures, and (3) transfer capability. In particular, two measures of security are considered, one for voltage security and another for flow security expressed with the following indices, respectively:

$$J_v = \sum w_i [(V_i - V_{is})/(V_{imax} - V_{imin})/2]^{2n}$$

$$J_p = \sum w_\ell (T_\ell / T_{\ell max})^{2n}$$

where

V_{imax}, V_{imin} is the maximum and minimum allowable voltage at bus i

V_{is} $= \frac{1}{2}(V_{imax} + V_{imin})$

V_i is the actual bus i voltage magnitude

T_ℓ is the actual circuit ℓ power flow

$T_{\ell max}$ is the circuit rating

w_i, w_ℓ are weight factors

J_v is a voltage security index

J_p is a flow security index.

The sensitivity analysis comprises three components. The first component is the optimization analysis, the second component is the sensitivity analysis for a specific condition, and the third component is the simulation process which may be based on Monte Carlo simulation or the enumerative approach. The objective of the optimization analysis is to optimize the operation of the system for a specific load condition. Next the sensitivity analysis computes the sensitivity of the power system attributes (operating cost, security, and transfer capability) with respect to FACTS elements parameters. This analysis provides useful information but only at the specified conditions. To obtain the impact of FACTS elements over a representative range of power system operating conditions, a Monte Carlo simulation or the enumerative approach is used to provide the statistical distribution of the information obtained from the optimization/sensitivity analysis. The overall procedure is embedded in the Monte Carlo simulation as it is illustrated in Figure 4.

6. SUMMARY AND CONCLUSIONS

Modeling, optimization, and simulation issues have been discussed pertinent to expansion plan evaluation methodologies. Methods have been proposed which favorably address the needs as they are shaped by recent trends in the electric power industry. Specifically, an enhanced model of the electric load is proposed which addresses the following concerns:

1. Power wheeling schedules
2. Customer owned generation (cogeneration, etc.)
3. Electric load modulation due to specific rate structures.

A new power flow/optimization formulation is proposed which favorably addresses the needs of a power system with increased number of controllable devices (FACTS). A sensitivity analysis embedded in a composite power simulation method provides measures of controllable device effectiveness on any pertinent power system attribute such as operating cost, security, and transmission losses.

Composite power system simulation methods have been reviewed. The enumerative approach and Monte Carlo simulation are the well developed methods. However, both fo these methods face serious practical limitations due to increase uncertainty in electric load and availability of non-utility generation. A promising approach has been recently introduced based on the probabilistic simulation method. The method provides the probability distribution function of specific system attributes by incorporating major operating practices. Much work needs to be done to incorporate transmission constraints and optimization procedures.

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BIOSKETCHES

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APPENDIX H: Probabilistic Analysis and Control of a less Regulated Power System

A. P. Sakis Meliopoulos, Feng Xia, and Xing Yong Chao, 'Probabilistic Analysis and Control of a less Regulated Power System', Presented at the NSF Workshop on Impact of a Less Regulated Utility Environment on Power System Control and Security, University of Wisconsin, Madison, April 19-20, 1991.

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Impact of a Less Regulated Utility Environment
on Power System Control and Security

Panel Session III

Probability and Game Theoretic Tools for
Analysis and Control in a Less Regulated Power System

Probabilistic Analysis and Control of a Less Regulated Power System

by
A. P. Sakis Meliopoulos, Feng Xia, and Xing Yong Chao

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Probabilistic Analysis and Control of a Less Regulated Power System

A. P. Sakis Meliopoulos, Feng Xia, and Xing Yong Chao

Abstract

Certain forms of competition and deregulation have already infiltrated the electric power system. The trend is towards increased competition and deregulation. These trends coupled with the nature and complexity of the electric power industry impose serious institutional and technical issues. The technical issues are overwhelming. Conventional approaches to power system analysis and control are clearly inadequate. This paper proposes new tools for power system analysis and controls which are suitable to study the effects of increased deregulation and competition. The proposed methods are extensions/reformulation of conventional analysis and control tools and can be easily implemented.

1. INTRODUCTION

The electric power industry is undergoing unprecedented changes. Technology advances, the PURPA legislation, and an increasingly competitive market are transforming the traditional ways of operating and planning an electric power system. From the operations planning point of view, the following major changes have occurred:

1. The uncertainty associated with system parameters has increased.

2. The electric load is increasingly becoming dynamic due to

    * distribution automation
    * load management
    * customer generation
    * Wheeling Load

3. Electric utilities seek to optimize their operating cost. The presence of independent power producers, qualified facilities, load management options, etc., affect the operational economics of utilities.

4. The electric power utility is changing its approach to marketing following a more aggressive approach in which value based marketing is central.

5. Customers are increasingly aware of cost and try to adopt to the changes by optimizing their operation.

In this environment, the dynamics of the system have increased with local controls and customer controls being a major player in determining system response. What is remarkable in this environment is the feeling that there are opportunities in the short term and potentially big problems in the long run.

The impact of deregulation and competition is qualitatively represented in Table 1.

The challenges we are facing are two fold: (a) To provide new models and analytical tools for evaluation of new scenarios which capture the basic phenomena of the changing environment. These tools will allow proper evaluation of alternatives and help adopt policies to benefit all, and (b) To provide new models and tools for controlling the real time operation of a changing electric power system. Obviously, the thesis here is that conventional tools are not adequate to handle the emerging needs.

This paper discusses a proposed model of the non-utility system consisting of electric loads, independent power producers, wheeling customer, etc. Several scenarios of non-utility system control are discussed. Methods of probabilistic analysis are delineated and the need for a comprehensive simulation tool for power system operation is postulated. Such a tool is being developed and described in this paper.

Table 1. Effects of Deregulation and Competition

Control	Decentralized	Centralized
System Response Dynamics	Unfavorable	Favorable
Cost	Low	High
Benefits	Short Term	Long Term
Complexity	Low	High

2. THE NON-UTILITY MODEL

Non-utility systems presently consist of electric loads and generation. Specifically, the non-utility system can be classified into:

* Conventional Electric Load
* Interruptible Load
* Shiftable Load
* IPP with Load
* Wheeling Load

In the presence of an active non-utility system, the old approach in which the electric load was an exogenous, independent quantity is not valid anymore. What makes this situation more complex is the fact that present day non-utility systems may be controlled by systems which use varying degree of feedback from the electric utility. The control schemes can vary from completely decentralized to totally centralized. The extreme cases are depicted in Figure 1 and 2.

In this environment it is necessary to decompose the electric load into (a) an exogenous process which is independent of the utility system and (b) a controllable component which is partially or totally controlled by the power system. It is further desirable to represent the exogenous process with a small numbers of independent variables. Such an abstraction is defined next.

The electric load can be represented as electric power injections at specified system buses. These injections are assumed to be stochastic processes. The use of ARIMA models is proposed for this purpose. The advantage of the ARIMA models are (1) they can accurately represent the periodic nature of power "wheeling" schedules, customer owned generation patterns, etc., and (2) they provide a good model to represent the uncertainties associated with customer owned generation, power "wheeling" schedules, etc.

The electric model is illustrated in Figure 3. It is a multiple input/multiple output ARIMA model. Specifically, the bus electric load, represented with the vector $S_L(t)$, is constructed from a vector of m independent white noise processes $\eta(t)$. Through an ARMA model, the independent white noise processes are coverted into a vector of stationary stochastic processes z(t). The vector z(t) is inverted to provide a vector of nonstationary stochastic processes x(t). Finally, the vector x(t) is translated into bus electric loads with the linear system L. The described model is represented with the following equations:

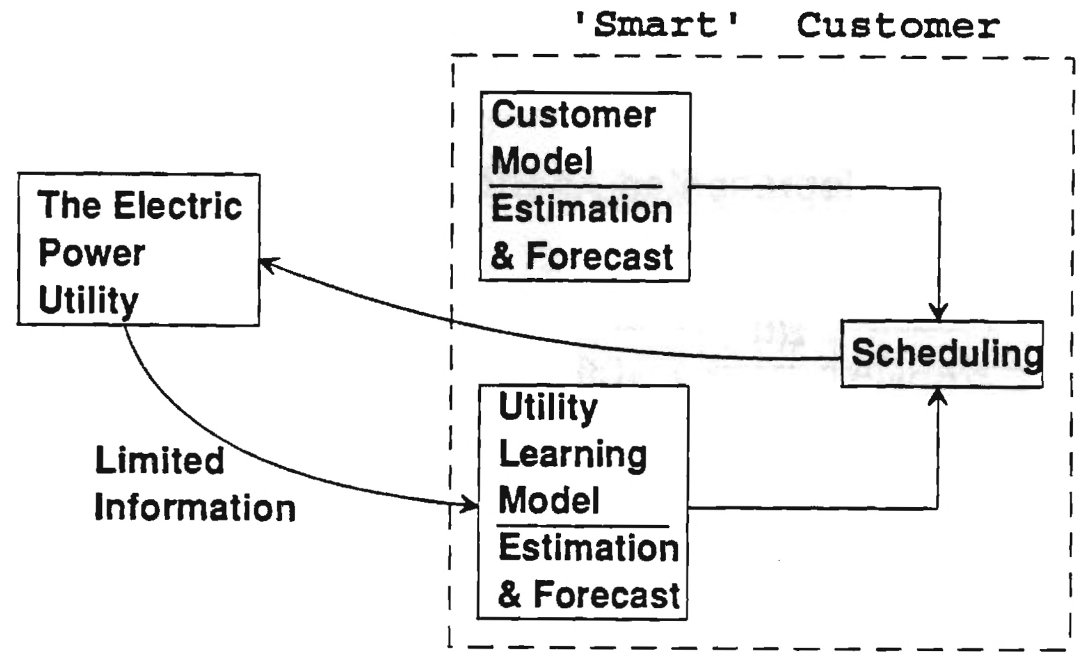


Figure 1. Completely Decentralized Control

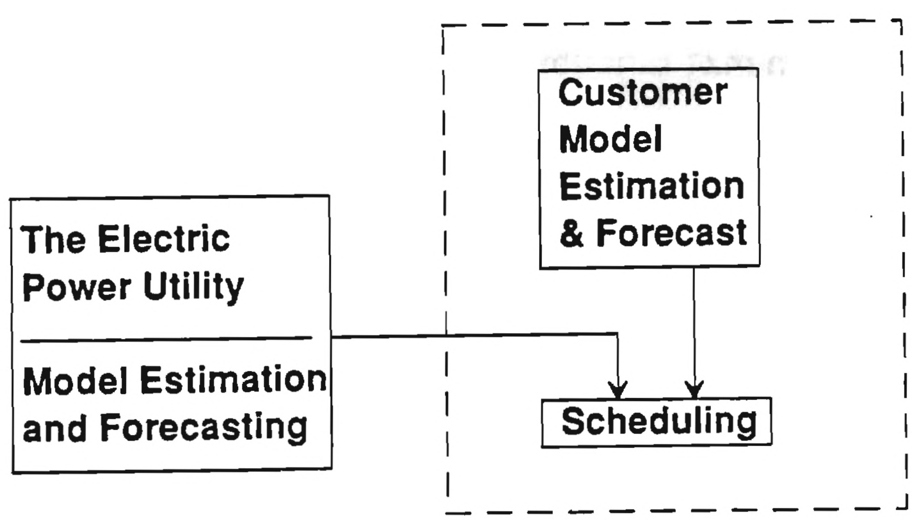
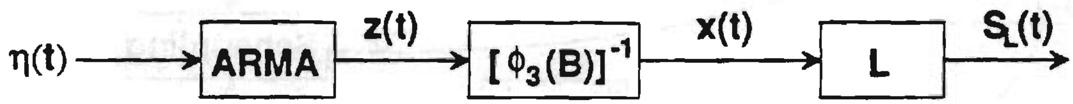


Figure 2. Totally Centralized Control

A. Electric Load Represented As Stochastic Process:



B. Electric Load Represented As Random Variable:

$$S_L = P_o + P\,v$$

P_o an nx1 vector
P an nxm matrix
v an mx1 random variable

Figure 3. Proposed Electric Load Model

$$\Phi_1(B)z(t) = \Phi_2(B)\eta(t) \tag{1}$$

$$\Phi_3(B)z(t) = x(t) \tag{2}$$

$$S_L(t) = Lx(t) \tag{3}$$

Where

$\eta(t)$	is an m-vector of independent white noise processes
$z(t)$	is an m-vector of stationary stochastic processes
$x(t)$	is an m-vector of nonstationary stochastic processes
$S_L(t)$	is an n-vector of bus electric loads
$\Phi_1(.),\Phi_2(.),\Phi_3(.)$	are vectors of arbitrary polynomials of the argument
B	is the backward operator
L	is an nxm matrix

ARIMA models have been extensively used to represent the electric load. It is well known that they are capable of representing the periodicities as well as the nonstationary property of electric load. The innovation introduced here is the linear model L which translates the low order nonstationary stochastic process vector $x(t)$ into the high order vector $S_L(t)$ of the bus electric loads. This innovation is justified on the basis that bus electric loads are typically strongly correlated. It is, therefore, reasonable to assume that they are generated as a linear combination of a small number of independent stochastic processes.

The optimal order of the ARIMA model (order of functions, Φ_1, Φ_2 and Φ_3) and the number of independent white noise processes (vector $\eta(t)$) is system dependent. Historical data of hourly bus electric loads can be utilized to identify the optimal order and the parameters of the described electric load model. For the purpose of this project, a simplification of this model will be utilized. The equations (1), (2) and (3) can be combined to yield:

$$S_L(t) = L\ \Phi_3(B)\ \Phi_1^{-1}(B)\ \Phi2(B)\ \eta(t) \tag{4}$$

It is well known the above equation can be approximated with a finite order polynomial of the backward operator B (moving average model). This approximation yields

$$S_L(t) = P_0 + \sum_{i=0}^{N} L_i \eta(t-i) \tag{5}$$

A further simplification is to consider the electric load at a specified time interval. In this case the stochastic processes $S_L(t)$ and $\eta(t)$ become random variables (independent of time). Equation (5) then becomes

$$S_L = P_0 + P_1 v \tag{6}$$

where

S_L n-vector of random variables representing the bus electric loads

v m-vector of random variables

P_0, P_1 appropriately dimensioned vector and matrix.

3. PROBABILISTIC METHODS FOR POWER SYSTEM ANALYSIS AND CONTROL

Probabilistic methods have been long ago recognized to be the premier tool for power system operations and expansion planning. In the presence of increased deregulation and competition, uncertainty increases and probabilistic methods become a must. In the past, probabilistic methods were applied separately to the generation and transmission system. Past design practices of a power system and its operating practices justified this decomposition. Presently however, and in the future, this decomposition is not justified. Recent trends have resulted in transmission constrained system which means that there is substantial impact on reliability, security, and cost due to the interaction between generation and transmission systems. The interaction cannot be ignored anymore, thus the need for composite power system simulation methods.

Efforts to develop composite power system simulation methods date at least 15 years ago. Three distinct approaches are identified in composite power system simulation:

1. Monte Carlo simulation
2. Analytic (Probabilistic Power Flow)
3. The enumerative approach / Minimal Cut States

Description and discussion of these approaches follows.

3.1 Monte Carlo Simulation

Monte Carlo simulation is easily implementable and provides a great tool for valuation of other methods. It is imperative that the Monte Carlo simulation be based on comprehensive models of electric load, generation, transmission, etc. In this way the results of the Monte Carlo method are directly comparable to the results of the proposed simulation method. Note that in the Monte Carlo method, the number of trials must be large for meaningful results. This requirement hinders the applicability of this method to large scale power systems. The Monte Carlo simulation is typically limited to small or medium size power systems.

The basic Monte Carlo approach can be applied for each hour in a year in chronological order (sequential approach) or the hours of the study time can be considered at random (random approach). The method requires a stochastic model of the electric load and a stochastic model of component availability. The models can be simple or sophisticated (inclusion of weather effects, econometric parameters, etc.). The simulation of the randomly selected system conditions is done with the use of power system analysis methods, such as load flows, dispatch algorithms, optimization algorithms, and model simulating operating policies, The result of the simulation are probability distribution functions of power system output variables (circuit flows, voltage levels, energy curtailment, etc.). These results are utilized in the computation of appropriate production quantities and reliability indices. The key issues in this approach are: (1) the number of trials must be large enough to adequately capture all possibilities of adverse effects on the system, and (2) the analysis of the effects for a specific trial must be as close to real world as possible and as efficient as possible.

The Monte Carlo approach has several advantages and disadvantages. Consider, for example, the application of this approach for reliability assessment. For meaningful results, it is imperative that a sufficiently large number of problematic system conditions be captured and simulated. However, the majority of the selected trials are problem free. This implies that the Monte Carlo method calls for the expenditure of considerable computing time in order to obtain sufficient confidence in the results. On the other hand, the method allows the analysis of complex systems without forcing the system model to become unrealistic. In addition, it offers a synthesis of the final results and a detailed description of the events that caused the results.

Conventional power flow analysis and contingency evaluation methods, lack the sophistication to accurately simulate the actual operation of the system. For example, power flow/contingency analysis may fail to converge in which case, the sample must be discarded. This results in biased results. Thus the need for a comprehensive tool for simulation of power system operation exist in a Monte Carlo approach.

Computational requirements of the Monte Carlo method pose practical limitations. Development of variance reduction methods and importance sampling methods provide promise to substantially reduce computational requirement when coupled with proper analytical models. The effectiveness of these techniques is dependent upon the rigor of the analytical method. This again suggests the need for a comprehensive tool for simulation of power operation.

3.2 Analytic (Probabilistic Power Flow)

Several analytic methods are available. Here we are interested in analytic methods for the composite power system. These methods involve the utilization of the probabilistic model for the generation and transmission system which are coupled with a probabilistic power flow method. The overall procedure is described as follows:

Consider the proposed stochastic load model and Markov model of generating unit availability. Further, consider the operation of the system during a specified period of time. This period of time may be contiguous and noncontiguous (for example, 1 pm to 3 pm each day for a period of one year). Utilizing the proposed electric load model, the electric load of a bus, during the period of simulation, can be characterized as a random variable with a probability distribution function and correlation to other bus loads. The total electric load can be characterized as a random variable, L, with a probability distribution function, $F_L(\ell)$. Similarly, utilizing the Markov model of generating units, the unit availability is represented as a discrete random variable. Assume that the n units of the system operate at level x_1, x_2, ..., x_n, respectively, while the total electric load is ℓ. If unit k is not in operation , then obviously x_k equals to 0. Since there is a finite probability that any unit can be forced out, the output of unit j, x_j, is considered to be a random variable with the probability of forced outage equal to q_j. We write

$$\begin{aligned} Pr(A_j = x_j) &= 1 - q_j, \qquad x_j \neq 0 \\ Pr(A_j = 0) &= q_j \end{aligned}$$

where A_j is a random variable representing the available capacity of unit j, q_j is the probability of unavailability that the output generator j is x_j equals $1-q_j$, and the probability that the same output is zero equals q_j.

For the condition that has been considered, the apparent load ℓ_a will be

$$\ell_a = 1 - x_1 - x_2 - .. - x_n \tag{7}$$

Since ℓ, x_1, x_2, ..., x_n, are not deterministically known, the above equation can be replaced with its equivalent equation in terms of the corresponding random variables

$$L_a = L - A_1 - A_2 - \ldots - A_n$$

where L is a random variable representing the total electric load and A_i is a random variable representing the output of unit i. Since the probability distribution function of the random variable L, A_1, A_2, ..., A_n are known and since these random variables are independent, the probability distribution function of the random variable La is easily computed with a series of convolutions [9].

If we assume that $\ell_a > 0$ (that is, load exceeds generation), then another unit should be brought into operation or one or more of the operating units should increase their output. Assume that unit i is operating at x_i and that it is selected according to a criterion to respond to any increases in the load. We shall refer to this criterion as the dispatch criterion. It is defined as to satisfy operational practices and constraints. The dispatch criterion can be arbitrary but usually is assumed to be the unit incremental cost. Without loss of generality, x_i may be equal to zero. In general, if $\ell_a > 0$, the output of unit 1 will increase from x_i to $x_i + \Delta x_i$, where Δx_i is a small increment (1-2 MW). We shall refer to this increment as the block Δx_i. The described formulation and direct application of probability theory yields expressions for the expected energy, cost of operation, required fuel, etc., from the Δx_i increase in the output of generator i, The detailed mathematical formulation is given in [23]

Upon completion of the simulation algorithm, the probability that unit i operates at level x_i or the probability density function of power injection at the generation buses as well as the joint probability density of any generating unit pair has been constructed. Thus, the power injections to the electric power network are characterized as random variables with known probability distribution functions and correlations. This is illustrated in Figure 4. It should be emphasized that the probability distribution functions at generation buses cannot be approximated with Gaussian distributions. This basic result can be also provide the performance parameters for each generating unit, such as (1) expected time of operation, (2) expected produced energy, (3) expected production cost, etc. [23].

The proposed method is also capable of simulating the operating practices and constrains of a power system. This objective is achieved by appropriate selection of the dispatch criterion mentioned in the description of the method. Specifically, the dispatch criterion is illustrated in Figure 5. The proposed method can accommodate an arbitrary dispatch criterion. The parameters of the penalty function are selected with a probabilistic optimization method which is described later. The solution of the optimization problem provides the parameters of the penalty function, such as the operational practices and constraints will be satisfied with maximum probability. Note that if the nonlinear penalty function is neglected, then the dispatch

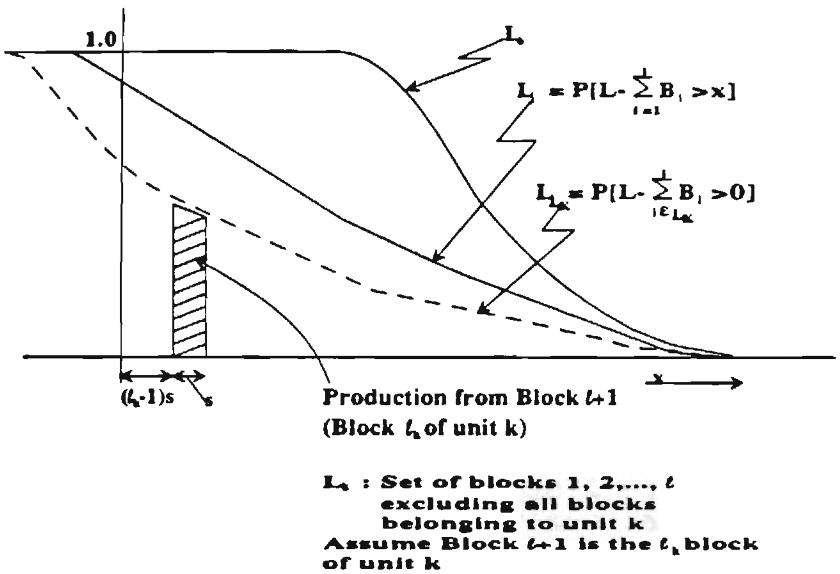


Figure 4. Analytic Method - Basic Concept

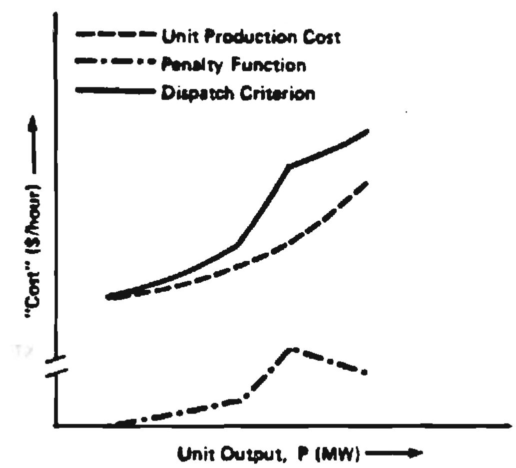


Figure 5 Illustration for the Definition of the Penalty Function and the Dispatch Criterion

criterion equals the actual operating cost of the generating unit. This selection amounts to simulating the economic dispatch only, neglecting other operating practices, such as interchange schedules, etc., and operating constraints, such as hydro energy limitations, transmission limitations, etc.

Once the power injections at the system buses have been probabilistically characterized with the above method, the probability density function of system output variables such as circuit loading, bus voltages, and system losses can be computed. This transformation can be achieved with any desirable degree of accuracy. The probability density functions of system output variables is translated into reliability indices, total system losses, etc.

The probability distribution functions of output variables is computed by first expressing the output variable as a function of bus power injections:

$$v_{oi} = \bar{v}_{oi} + \sum_j f_{oij}(x_j - \bar{x}_j) \tag{9}$$

where

V_{oi} is the output variable of interest

$\bar{x}_j$ is the expected value of the power injections at bus j (x may represent real or reactive power)

x_j is the power injection at bus j

$f_{oij}(.)$ is, in general, a nonlinear function of the argument.

For practical calculations, the function $f_{oij}(.)$ is expressed as a piecewise linear function:

$$f_{oij}(x_j-\bar{x}_j) = \{s^k_{oij}y;\ x_j{}^{k-1}-\bar{x}_j \le y \le x_j{}^k-\bar{x}_j;\ k= 1,2,\ldots,\} \tag{10}$$

where

$s^k_{oij} = dv_{oi}/dx_j$ computed at $y = (x_j{}^{k-1} + x_j{}^k - 2\bar{x}_j)/2$

References [22] and [23] provide an efficient procedure for computing the sensitivities s^k_{oij} based on the costate (adjoint) equation.

Equation (10) states that the function $f_{oij}(.)$ can be considered linear in an interval $[x_j^{k-1} - \bar{x}_j, x_j^k - \bar{x}_j]$. The interval of validity of the linearized model depends on the parameters of the system in the neighborhood of bus j. A simple method for selecting the bounds $(x_j^{k-1} - \bar{x}_j)$ and $(x_j^k - \bar{x}_j)$ has been developed and reported in [22]. The method is based on the observation that the error is approximately an invariant function of the quantities $(P_{inj}/\Sigma Y)$ and $(Q_{inj}/\Sigma Y)$, where P_{inj} and Q_{inj} are real and reactive power injection at a bus and ΣY is the sum of admittances connected to the bus. For a given allowable error, these quantities must not exceed the values ε_1 and ε_2 which are typical selected to be 0.02-0.05. Subsequently, P_{inj} and Q_{inj} are translated into the bounds $(x_j^{k-1} - \bar{x}_j)$ and $(x_j^k - \bar{x}_j)$. It is important to note that the interval of validity of the linearized model is quite wide even for small allowable errors, i.e., 2%. Practically, this means that variations of bus electric loads can be accurately represented with only one segment of the piecewise linear equation (10). More than one segment is needed only at generation buses, or buses with very large load, where the power injection variations are high. The probability density function of the output variable v_{oi} can be computed from the probabililistic model of the power injection x_j and equations (9) and (10) with direct application of probability theory.

A difficulty associated with the described probabilistic simulation is the incorporation of transmission system optimization limitations. Specifically, probabilistic network models, such as the probabilistic power flow, have not incorporated operating constraints in the formulation. Recently, a method has been developed which is based on expressing operating constraints on a probabilistic basis and which imposes these constrains to probabilistic power flow. A concise description of this method follows. The proposed simulation method imposes the effects of transmission system constraints on the operation of the generation system with set of parameters expressing the dispatch criterion. This approach enables the decomposition of the generation system and transmission system simulation methods. The parameters of the dispatch criterion are determined with an optimization problem which optimizes the operation of the system subject to transmission and other operating constraints. the optimization problem is postulated as follows: Determine the required adjustments to the generation bus power injections, such that the probability that an operating constraint is violated is minimized. The operating constraints may be: (1) circuit loading constraints, (2) bus voltage allowable range, (3) net power interchange constraints, etc. The mathematical formulation of the probabilities optimization problem is presented next.

The formulation of the probabilistic optimization is based on modeling the probabilities operating constraints with deterministic constraints. As an example, consider the power flow T_k of circuit k:

$$T_k = g(x_j \; ; \; j = 1,2,\ldots) \tag{13}$$

where

x_j is the power injection at bus j.

Let $\overline{T}_k$ be the rating of circuit k. Then the probability of overload is defined as:

$Pr[T_k - \overline{T}_k \geq 0]$ = probability that circuit k is overloaded

Recall that the probability density function of the output variable T_k depend on the probability density function of the power injection variables x_j. For a mathematical formulation of the problem, define the following control variables:

- u_{Gj} is the vector of parameters of the penalty function of generating unit j.
- u_{Li} is the control parameter expressing load shedding at bus i.

Other control variables may be added if desirable. Then

$$Pr[T_k - \overline{T}_k \geq 0] = h_k(u_{Gj}, u_{Li} \; ; \; j = 1,2,\ldots) \tag{14}$$

The above modeling methodology is repeated to other operating constraints, such as interchange constraints, hydro energy constraints, bus voltage constraints, etc.

Subsequently, an optimization problem is defined which minimizes the deviation from economic operation subject to constraints that the probability of an operating constraint is violated is less than a prespecified value:

Minimize $\sum_j |u_{Gj}|$

Subject to

$$Pr[T_k - \overline{T}_k \geq 0] = h_k(u_{Gj}, u_{Li} \; ; \; j = 1,2,\ldots) \leq P_{ka}$$

$$Pr[E_{hl} - E_{hl} > 0] = h_l(u_{Gj}, u_{Li} ; j = 1,2,...) = 0$$

Etc.

The defined probabilistic optimization problem can be solved with linear programming techniques. For this purpose, probabilistic constraints are linearized with respect to the parameters u_{Gj}, u_{Li}.

3.3 The Enumerative Approach

The enumerative approach has been extensively used in North American for adequacy evaluation/reliability analysis. The basis idea is to enumerate all possible states of an electric power system, to analyze the state and store the results for subsequent processing. The conceptual and computational problems are serious. As an example, a very large number of multiple generating unit outages must be considered since the probability of these events is substantial (generating unit probability of unavailability is quit high). The problem of a priori determining the severity of multiple unit outages is a challenging problem. An effort toward this goal is the wind-chime method developed by PTI for EPRI. The method considers a number of contingency levels. At each level the "binding" contingences (states) are identified with a multiplicity of contingency ranking methods. The enumerative method is extremely time consuming.It is our strong belief that state enumeration methods are at best ineffective in addressing the present day needs for composite power system simulation (which is the basic tool for reliability analysis). On the other hand, they provide useful and complete information for the enumerated states. As the system becomes increasingly complex and stressed to the limits, the need for effective composite system simulation becomes relevant. Aggregate (macroscopic) simulation methods of the composite power system are needed.

An important ramification of the enumerative approach is the enumeration of the minimal cut states as it is illustrated in Figure 6. The idea there is to identify only the minimal cut states which represent only a small fraction of the total number of states. The central problem in this approach is the need for a tool with accurately will simulate the power system operation. Such a tool will accurately detect transition states which results to violation of a criterion.

In summary probabilistic analysis and control methods require as a basic tool and a method for accurate simulation of power system operation. A proposed method to fill this need is proposed next.

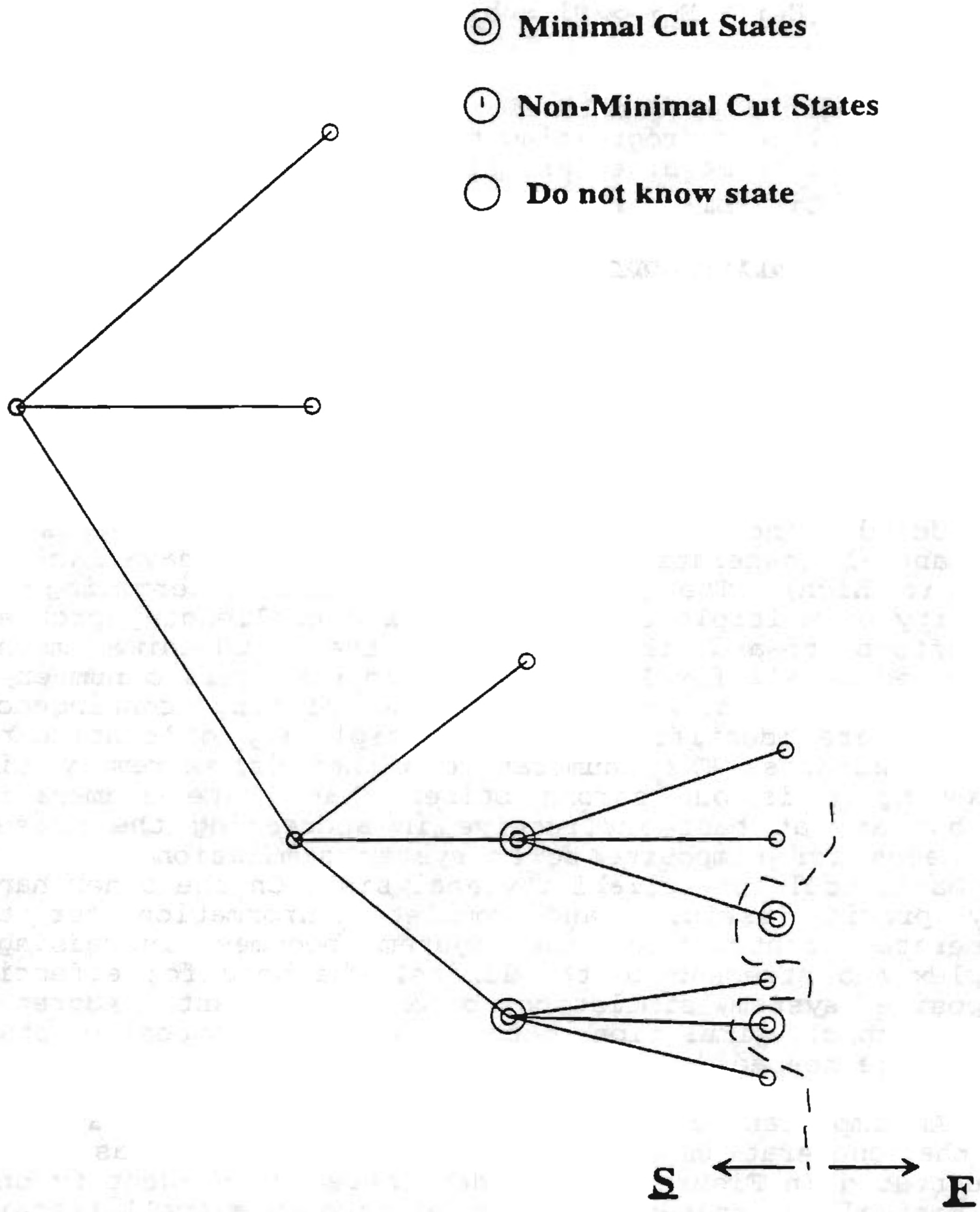


Figure 6. Illustration of Search Algorithm for Minimal Cut States

4. STATIC SIMULATION OF POWER SYSTEM OPERATION

The power system of the future will serve a complex non-utility system and will rely on FACTS elements to attain an acceptable operating condition. This means that standard power flow algorithms may not be adequate to address the needs. A new approach is proposed for power flow analysis which within its algorithm will be able to dispatch FACTS elements as necessary. Utilization of FACTS elements is viewed as remedial actions which are applied by an optimization model within the power flow solution algorithm. The approach accommodates the varying degree of centralized control in the power system of the future. The following formulation is proposed which achieves this goal.

Consider an electric power system comprising n buses. Let the state of the system be represented with the vector x (x contains bus voltage magnitude and phase in the usual sense). Let the vector u represent the available controls consisting of (a) generation bus voltage magnitude, (b) switchable capacitors or reactors, (c) load transfer, etc. Assume a given operating state x^o and settings of controls u^o. Further, consider bus i as is illustrated in Figure 7. Unless x^o and u^o represent a power flow solution, there will be a power mismatch at bus i equal to $P^o_{mi} + jQ^o_{mi}$. Now assume that a fictitious generating unit is placed at bus i as in Figure 7. Let the output of the fictitious unit at bus i be $P^o_{mi} + jQ^o_{mi}$. In this case x^o and u^o represent the present operating condition of the system. The actual operating condition of the system can be obtained by gradually reducing the output of the fictitious generating units to zero and computing the system variables x and u which will make P_{mi}, Q_{mi} equal to zero. This transition can be achieved along a trajectory which maintains feasibility and optimality. Mathematically, this procedure is formulated as an optimization problem as follows:

Minimize $\quad - \mu \Sigma_i (|dP_{mi}| + |dQ_{mi}|) + \Sigma_j f_j(P_{gj})$

Subject to
Power balance equation
Voltage constraints
Circuit flow constraints
Net MW interchange constraints
Unit reactive power output constraints
$- P^o_{mi} \leq dP_{mi} \leq 0 \quad , \quad i = 1, \ldots, n$
$- Q^o_{mi} \leq dQ_{mi} \leq 0 \quad , \quad i = 1, \ldots, n$
Other pertinent constraints

................(1)

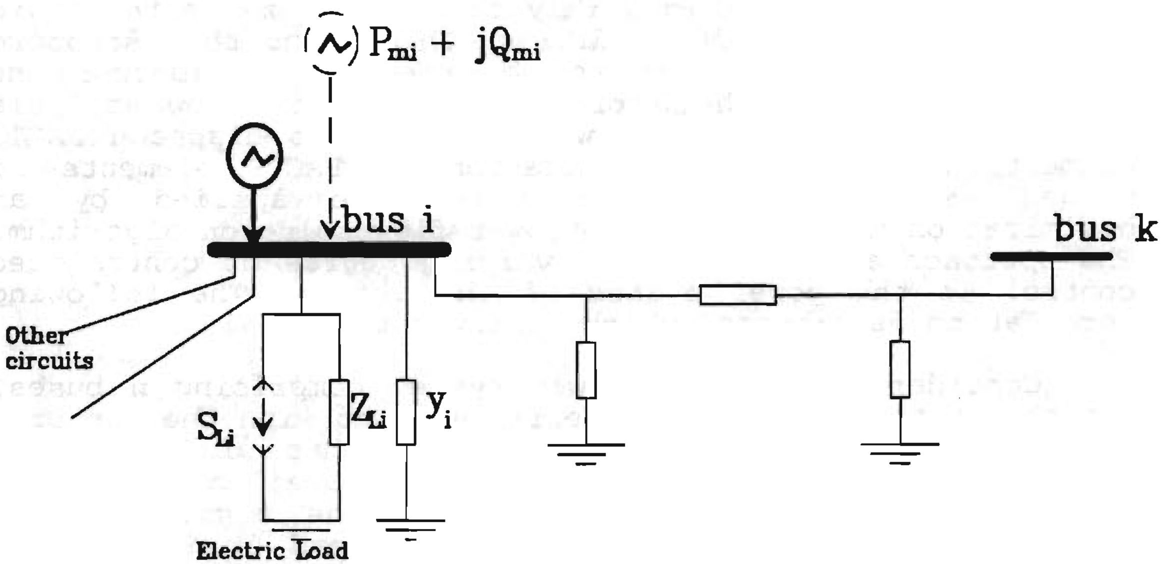


Figure 7. Illustration of a General Bus i of an Electric Power System

where $dP_{mi} = P_{mi} - P^o_{mi}$, $dQ_{mi} = Q_{mi} - Q^o_{mi}$, μ is an appropriately selected penalty function, and P_{gj} is the real power output of generating unit j. The first term of the objective function is a penalty function which tends to reduce the fictitious mismatches to zero, thus reaching feasibility. The second term of the objective function is a preselected function to be optimized. In the present case, this term expresses the total generation cost. However, it can be substituted with any other function of interest such as total losses, etc.

The defined optimization problem is a large scale problem. The size of this problem can be drastically reduced with simple transformations. Specifically observe that the mismatch variables can be substituted with one control variable v as follows

$$dP_{mi} = - P^o_{mi}\, v \qquad i = 1, 2, \ldots, n$$

$$dQ_{mi} = - Q^o_{mi}\, v \qquad i = 1, 2, \ldots, n$$

......(2)

where the variable v represents the normalized change of the mismatch variables ($0 \le v \le 1$). Note that this transformation replaces all the mismatch variables (a total of 2n) with a single variable, v.

The variables P_{gj} (a total of m, m is the number of generating units) can be also replaced with only two variables while implicitly incorporating the economic dispatch process. Specifically, consider the economic dispatch problem

$$\text{min.} \quad \Sigma_j\, f_j(P_{gj}) = \Sigma_j\, \alpha_j + \beta_j\, P_{gj} + \gamma_j\, P^2_{gj}$$

$$\text{S.t.} \quad \Sigma_j\, P_{gj} - q - P_L = 0$$

......(3)

where q is the total transmission system losses, P_L is the total system load

Assume that the present generation schedule is an economic dispatch schedule. This means that P^o_{gj}, j = 1, 2, ..., m is a solution of the above defined optimization problem, satisfying the following equations

$$\beta_j + 2\, \gamma_j\, P^o_{gj} - \lambda^o(1 - \frac{dq}{dP_{gj}}) = 0, \quad j=1,2,\ldots m$$

$$\Sigma_j\, P^o_{gj} - q^o - P^o_L = 0$$

Now assume a small change in the variable λ^o, $d\lambda$. This change will cause a change in the generating unit outputs and load as follows:

$$dP_{gj} = \begin{cases} \frac{1}{2\gamma_j}(1 - \frac{dq}{dP_{gj}})\, d\lambda & \text{if } \gamma_j \neq 0 \text{ and } P^{min}_{gj} \leq P^o_{gj} \leq P^{max}_{gj} \\ 0 & \text{otherwise} \end{cases}$$

$$dP_L = \Sigma_j \frac{1}{2\gamma_j}(1 - \frac{dq}{dP_{gj}})^2\, d\lambda$$

......(4)

Thus if the electric load increases by dP_L the unit j should increase by dP_{gj} above. The economic participation factor for unit j is

$$p_{fj} = \begin{cases} \dfrac{\frac{1}{2\gamma_j}(1 - \frac{dq}{dP_{gj}})}{\Sigma_k \frac{1}{2\gamma_k}(1 - \frac{dq}{dP_{gk}})^2} & \text{if } \gamma_j > 0 \text{ and } P^{min}_{gj} \leq P^o_{gj} \leq P^{max}_{gj} \\ 0 & \text{otherwise} \end{cases}$$

......(5)

In order to allow flexibility in the model, two economic participation factors are defined, one for generation increase and another for generation decrease. Let w_1 be a variable representing total generation increase and w_2 be a variable representing total generation decrease. In this case,

$$dP_{gj} = p^+_{fj}\, w_1 + p^-_{fj}\, w_2$$

......(6)

where p^+_{fj} is the economic participation factor for generation increase. It is equal to p_{fj} if $p^{min}_{fj} \leq p^o_{fj} < p^{max}_{fj}$ and equals zero if $p^o_{fj} = p^{max}_{fj}$.

p^-_{fj} is the economic participation factor for generation decrease. It is equal to p_{fj} if $p^{min}_{fj} < p^o_{fj} \leq p^{max}_{fj}$ and equals zero if $p^o_{fj} = p^{min}_{fj}$.

Above transformation reduces the variables P_{gj} , $j=1,2,...,m$ to only two variables w_1 and w_2.

Upon substitution of above transformations and linearization of above problem around the present operating point yields

$$\text{Min.} \quad -\mu\,\Sigma(|P^{o}_{mi}| + |Q^{o}_{mi}|)\,v \;+\; c_1\,w_1 + c_2\,w_2 + c^T\,du$$

$$\begin{aligned} \text{S.t.} \quad & a_1\,v + a_2\,w_1 + a_3\,w_2 + a_4^T\,du = b \\ & \text{Linearized voltage constraints} \\ & \text{Linearized circuit flow constraints} \\ & \text{Other linearized constraints} \\ & 0 \le v \le 1.0 \\ & 0 \le w_1 \le w_1^{max} \\ & 0 \le w_2 \le w_2^{max} \\ & du^{min} \le du \le du^{max} \end{aligned}$$

......(7)

where u are other available controls such as transformer taps, switched capacitors and inductors, etc

$$f(x,u) = f(x^{o},u^{o}) + c^T\,du$$

The linearized model is valid only in a small region around the operating point[10]. Define

$$y = \begin{pmatrix} v \\ w_1 \\ w_2 \\ du \end{pmatrix}$$

Then the region of validity may be expressed as follows:

$$y^{min} \le y \le y^{max}$$

The overall problem is stated as follows

$$\text{Minimize} \quad [-\mu\,\Sigma(|P^{o}_{mi}| + |Q^{o}_{mi}|),\ 0\ ,\ C^T\ ] * y$$

$$\text{Subject to} \quad A\,y = b$$

$$y^{min} \le y \le y^{max}$$

......(8)

Solution of this problem provides the new generation schedule, new settings of controls and new power mismatches. A nonzero value of power mismatches indicates that the algorithm has not converged yet. The computed solution is then implemented, the power flow solution is updated and the process is repeated until the variables dP_{mi}, dQ_{mi} become zero. If a solution cannot be found, load shedding is invoked as a remedial action. The overall algorithm is illustrated in Figure 8.

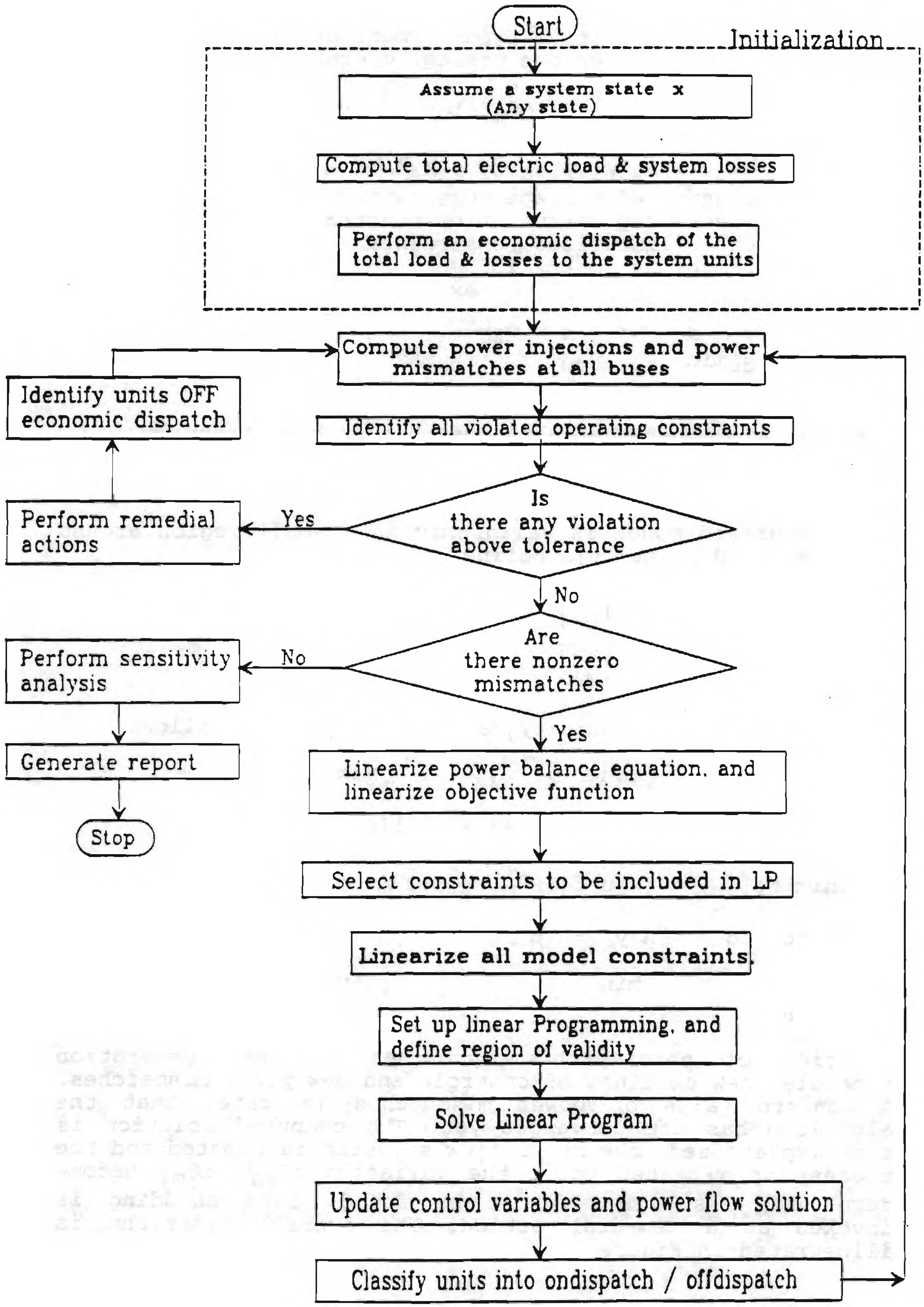


Figure 8. Flow Chart of the Static Simulation of Power System Operation

The proposed approach to the power flow problem has several advantages. First, it combines the remedial actions with the power flow solution and, thus, increases the efficiency of the overall model as compared to performing a power flow solution and then a remedial action separately. Second, it eliminates the necessity of adjustments during the power flow solutions such as net MW export adjustments, capacitor/reactor switching with local logic, etc. These adjustments may require several power flow iterations. Third, and most important, it will guarantee that several power mismatches will not result in nonconvergent power flows.

Efficiency-wise, the proposed power flow/optimization algorithm requires overall less execution time than the usual power flow with interchange adjustments, capacitor/ reactor switching, etc.

4.1 Model Linearization

At each major iteration of the algorithm the model is linearized and solved via a linear program. The linearization involves three distinct tasks: (a) Linearization of the objective function, (b) linearization of the power balance equation and (c) possible linearization of operating constraints which are included in the model presented in an earlier paper[10]. Here it will be shown that the linearization of the objective function and the power balance equation can be performed with only one costate vector solution, i.e. only one forward and back substitution. The linearization of the operating constraints requires one forward and back substitution per constraint.

4.1.1 Linearizing objective & power balance equation

For this purpose, linearized expression for the transmission losses is computed first. The general expression of the sensitivity of the transmission losses **q** with respect to a control variable is

$$\frac{dq}{du} = \frac{\delta q}{\delta u} - \hat{x}^T \frac{\delta g}{\delta u} \qquad \text{......(9)}$$

where u is the control variable of interest
g represents the power flow equations
$\hat{x}$ represents the solution of the adjoint network which is defined from

$$J^T \hat{x} = \frac{\delta q}{\delta x}$$

where $\frac{\delta q}{\delta x}$ is the partial derivative of the transmission loss q with respect to the variables x (bus voltage phase angles and magnitude)

J is the Jacobian matrix of the power flow equations

Above computation provides the derivatives $\frac{dq}{dP_{gi}}$ and thus the penalty factors $(1 - \frac{dq}{dP_{gi}})$. Subsequently, using equation (5) and (6) the economic participation factors p^{+}_{fj} and p^{-}_{fj} are computed. The linearized objective function is

$$\Sigma_j \; f_j(P^o_{gj}) + w_1 \, \Sigma_j \frac{\alpha_j + \beta_j \; P^o_{gj}}{p^{+}_{fj}} + w_2 \, \Sigma_j \frac{\alpha_j + \beta_j \; P^o_{gj}}{p^{-}_{fj}}$$

......(10)

The linearized power balance equation is

$$a_1 v + a_2 w_1 + a_3 w_2 + a_4^T u = b$$

......(11)

where
$$a_1 = \Sigma_{i,j} \; (\hat{x}_i P^o_{mi} + \hat{x}_j Q^o_{mj})$$
$$a_2 = \hat{x}_i \; p^{+}_{fi}$$
$$a_3 = \hat{x}_i \; p^{-}_{fi}$$

4.2 Method Evaluation

The proposed method has been tested with several power systems. The results of the testing with two systems will be presented: (1) the IEEE Reliability Test System (24 buses), and (2) the Georgia Power Company bulk power system (1304 buses, 1546 circuits, 117 transformers [33 variable tapes], 81 capacitor banks, 137 generating units).

Convergence performance for the IEEE 24 bus Reliability Test System are illustrated in Table 2. The table lists for each major iteration: (1) type of iteration. OPF means optimal power flow iteration and RA means remedial action iteration; (2) the total number of constraints and the number of constraints include in the LP model, (3) the maximum real and reactive power mismatch; (4) the total generation cost; and (5) the total transmission loss. This table clearly illustrates the essence of the algorithm. It

Table 2. Convergence Performance of the Proposed Method 24 Bus RTS System

Iteration #/Type	Constraints Total/in LP	Max Power Mismatch Real/Reactive (in pu)	Total Generation Cost ($/hr)	Total System Loss(pu)
1 / OPF	1 / 1	8.0000 / 1.4595	45148.83	0.0337
2 / OPF	1 / 1	6.7346 / 1.2068	45285.06	0.1334
3 / OPF	1 / 1	5.4717 / 0.9597	45613.51	0.2993
4 / OPF	1 / 1	4.2109 / 0.7183	46149.63	0.5339
5 / OPF	1 / 1	2.9517 / 0.4837	46927.08	0.8435
6 / OPF	1 / 1	1.6936 / 0.2579	48027.92	1.2423
7 / RA	7 / 5	1.6936 / 0.2579	48188.80	1.1027
8 / RA	8 / 6	0.6936 / 0.1579	48356.33	1.0580
9 / OPF	8 / 6	0.0952 / 0.0642	49844.01	0.9928
10 / OPF	8 / 6	0.0083 / 0.0100	50289.72	0.9878

Table 3. Convergence Performance of the Proposed Method 1304 Bus System

Iteration #/Type	Constraints Total/in LP	Max Power Mismatch Real/Reactive (in pu)	Total Generation Cost ($/hr)	Total System Loss(pu)
1 / OPF	1 / 1	33.4200 / 3.7840	319430.87	0.4488
2 / OPF	1 / 1	23.3997 / 2.7741	322634.50	1.8802
3 / RA	10 / 3	23.3997 / 2.7741	322634.50	1.8862
4 / OPF	10 / 3	13.3449 / 1.5822	329629.50	3.9068
5 / OPF	10 / 3	3.3713 / 0.5959	335517.90	4.8088
6 / OPF	10 / 3	0.0842 / 0.1029	335597.75	4.7560
7 / RA	11 / 4	0.0129 / 0.0155	335767.65	4.7998
8 / OPF	11 / 4	0.0074 / 0.0096	335767.65	4.7589

starts from an optimal solution with practically unloaded network, minimal losses, and minimal total loaded, the losses increase, and the total generation cost increase. At the solution, the mismatches go to zero, the total losses are 98.78 MW and the generation cost is 50289.72 \$/hr. Note that the algorithm twice because of encountered infeasibility (violated operating constraints). It is interesting to compare this solution to the usual power flow solution. The usual power flow solution yields:

Total Generation Cost:	53380.664 \$/hr
Total Transmission Losses:	56.654 MW

Note that losses are lower but cost is much higher.

Convergence performance for the Georgia Power Company 1304 bus system are illustrated in Table 3. The same format as Table 1 is used. Note that the algorithm switches to remedial actions twice for this system. The optimal solution is:

Total Generation Cost:	335767.65 \$/hr
Total Transmission Losses:	475.89 MW

The usual power flow solution with a generation dispatch specified by Georgia Power Company is:

Total Generation Cost:	400460.625 \$/hr
Total Transmission Losses:	406.39 MW

Performance evaluation is given in Table 4 for the Georgia Power Company system (1304 bus system). The table provides for each iteration the number of total constrains and the number of constraints included in the LP model, and the execution times of three major components of the algorithm, i.e. model linearization, LP setup and solution, and power flow update. The power flow update timing includes the recomputation and refactorization of the Jacobian matrix. The table also includes the total execution time for each major iteration. It should be clear that the total execution time is the sum of the numbers in the last column of Table 3. Note that the total execution time of the nondivergent/optimal power flow is 79.87 seconds on a IBM PS/2 model 70 computer.

4.3 Computation of Probability Distribution

The proposed formulation provides an efficient algorithm for computation of probability distributions of specific attributes such as (a) total operating lost, (b) security indices, etc. For this purpose consider an attribute denoted with the function a(x,u). As controls u can be considered to be the independent random variables v which describe the electric load model. Upon linearization of the attribute a(x,v) we obtain

Table 4. Performance Evaluation of the Proposed Method 1304 Bus System/Times Are in Seconds on a PS/2, Model 70, 25 Mhz PC

Iteration #/Type	Constraints Total/in LP	Model Linearztn	LP Setup and Solution	Power Flow Update	Total Execution Time
1 / OPF	1 / 1	0.87	0.93	5.43	7.63
2 / OPF	1 / 1	0.54	2.30	5.54	8.68
3 / RA	10 / 3	1.64	0.82	5.43	8.68
4 / OPF	10 / 3	1.12	3.62	5.43	10.13
5 / OPF	10 / 3	1.09	3.90	5.49	10.76
6 / OPF	10 / 3	1.09	3.57	5.49	10.43
7 / RA	11 / 4	1.64	4.94	5.49	12.80
8 / OPF	11 / 4	2.08	2.91	5.49	10.76

$$a(x,u) = a(x^0,v^0) + \sum_i s_i dv_i \qquad \text{(i)}$$

The coefficients s_i are computed with

$$s_i = \frac{\delta a(x,v)}{\delta v_i} - \hat{x}^T \frac{\delta g(x,v)}{\delta v_i}$$

$$J^T \hat{x} = \frac{\delta a(x,v)}{\delta x_i}$$

$$J = \text{Jacobian}$$

The above linearized equation can be computed with practically one forward and back substitution on the vector $\delta a(x,v)/\delta x_i$ and some additional minor computations. The probability distribution function of the attribute $a(x,v)$ is computed with a series of convolutions defined with equation (i) since the variable v_i are independent random variables.

5. SUMMARY

This paper presented a proposed modeling approach for non-utility systems and reviewed several probabilistic analysis and control tools suitable for electric power system with increased controllability and uncertainty. A specific model for static simulation of power operation has been discussed.

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BIOSKETCHES

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APPENDIX I: Non-Divergent and Optimal Power Flows: A Unified Approach

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NON-DIVERGENT AND OPTIMAL POWER FLOWS: A UNIFIED APPROACH

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ABSTRACT: This paper describes a unified framework for the formulation and solution of the non-divergent and optimal power flow. The formulation and solution are based on mathematical programming techniques that incorporate the process of economic dispatch, observe the operating constraints, and guarantee the convergence of power flow. The resulting solution has the properties of the usual optimal power flow, i.e. it optimizes a predefined objective function.

KEYWORDS: Optimal Power Flow, Nondivergent Power Flow, Economic Dispatch, Remedial Actions

1. INTRODUCTION

Optimal power flow (OPF) has undergone intensive research and development over the past twenty years [1-9]. Today, production grade OPF codes are available. However, some serious deficiencies still exist which limit their scope of application and practical value [7,8]. Some of the reasons are (1) real-life OPF problems are much more complicated than their classical formulation, (2) present trends in the electric power industry, such as integration of FACTS elements, have affected operation and optimization practices, and (3) implementation-wise, the speed of present OPF solution is problematic with respect to power system operation. Reformulation of the OPF problem to accommodate the new developments in power system modeling and optimization, and solution methods to alleviate these deficiencies are necessary.

Basic optimization approaches applied to the OPF problem, can be roughly divided into those based on successive linear programming [3,9], successive quadratic programming (QP) [4], and nonlinear programming [1,3,8]. As an example in Reference 4, the OPF problem is approximated by a quadratic objective and sparse linearized constraints expressed in terms of deviations from the current operating state. The resulting problem is solved for the control variable corrections by quadratic programming. This process is iterated to convergence with infeasibility being easily detected. However, the method requires high computational effort, and it is difficult to incorporate discrete controls. Computational effort tends to rise rapidly with the number of controls and constraints. Successive linear programming based methods have been proven to provide fast optimal power flow solutions [3,9]. Recently, interior point methods have been introduced and research activities are under way to apply these methods to the optimal power flow.

It has been recognized that the optimal power flow reflects more realistically the actual operation of a power system. Yet in many applications, i.e. planning, reliability analysis, etc., the conventional power flow is utilized because of the superior efficiency of the conventional power flow as compared to the optimal power flow. On the other hand, the usual formulation of the power flow problem may lead to nonconvergence or it may converge to an undesirable solution, such as the slack bus may not have enough generation to supply the required generation. This situation occurs mainly because the traditional formulation of the power flow requires the preselection of the PQ, PV, and slack buses and prespecification of the controls of the system. A diverged power flow solution may be avoided by adjusting the controls such as transformer taps, generation schedule, generation bus voltage levels, etc. The OPF approach provides for adjustments of the controls.

A unified formulation and solution method for the optimal power flow problem is proposed which implicitly incorporates the process of economic dispatch, observes the operating constraints, and guarantees convergence if a solution exists. The resulting dispatch satisfies the operating constraints and minimizes the operating cost, as the usual optimal power flow.

2. MOTIVATION

Traditionally, the optimal power flow is solved as follows: First, a power flow solution is obtained assuming a generation schedule which is near the economic dispatch solution. In order to solve the power flow problem, it is necessary to specify in advance the generating unit outputs except the slack bus. The output of the units at the slack bus is determined after the power flow solution. In this approach, it is possible that one or more undesirable conditions may occur: (1) the output of the slack bus generation may be outside the physical limits of the units (infeasibility), (2) the power flow algorithm may not converge, (3) the solution may move far from the optimal solution, and (4) many operating constraints may be violated. In case of a successful power flow solution, a constrained optimization procedure is applied to optimize the operating point which at the same time satisfies the operating constraints. This procedure is pictorially illustrated in Figure 1. Assume that the initial generation schedule of a two generating unit system is point A in Figure 1a. The power flow solution is decided by point B assuming that P_{g1} is the slack bus generation. The length of the excursion AB is equal to the amount by which the system losses were mispredicted. Note that the operating point B has moved into the infeasible region. Upon optimization, the operating point C is obtained which is the optimal power flow solution. In this qualitative illustration, it is apparent that the usual approach moves from point A to point B, then to C. Obviously, many computations can be avoided if the solution can move directly from A to C.

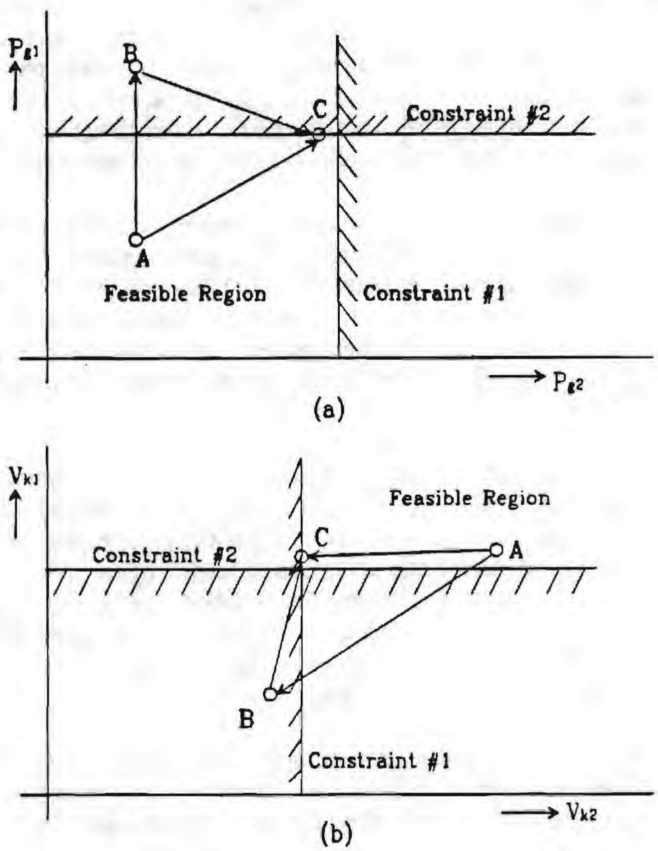

Figure 1 Illustration of Power Flow Solution
(a) Correction of Unit Output is required
(b) Correction of Bus Voltage is required

Contingency solution for security analysis or reliability assessment typically involves a post contingency power flow solution followed by application of remedial actions to alleviate violated operating constraints. There are two potential problems with this approach: (1) the post contingency power flow algorithm may not converge because the system is severely stressed, and (2) separation of the power flow solution and remedial action algorithm leads to unnecessarily large computational effort. The traditional approach to separate the power flow algorithm from the remedial action algorithm may lead to a situation in which a solution may not be found even if a solution may exist by application of appropriate remedial actions. This is quantitatively illustrated in Figure 1b. Assume that the precontingency operating condition is represented with point A. the axes have been selected to be voltage magnitudes at two buses of the system. A possible solution of the post contingency power flow may be represented with point B which shows under-voltage at both buses. Application of remedial actions may bring the operating condition to point C. It must be apparent that efficiency-wise, it is expedient to develop a method which will move from point A directly to point C rather than going through point B.

This paper presents a new formulation of the power flow problem which combines the traditional power flow, remedial actions, and optimal power flow in one unified approach. The new formulation naturally leads to a nondivergent power flow algorithm.

3. PROPOSED PROBLEM FORMULATION

Consider an electric power system comprising n buses. Let the state of the system be represented with the vector x (x comprises bus voltage magnitude and phase in the usual sense). Let the vector u represent the available controls consisting of (1) generation bus voltage magnitudes, (2) transformer taps, (3) transformer phase shift adjustments, (4) switchable capacitors or reactors, (5) load transfer, and (6) interchange adjustments. Assume a given operating state x^o and settings of controls u^o. Further, consider bus i as is illustrated in Figure 2. Unless x^o and u^o represent a power flow solution, there will be a power mismatch at bus i equal to $P^o_{mi} + jQ^o_{mi}$. Now assume that a fictitious generating unit is placed at bus i as in Figure 2. Let the output of the fictitious unit at bus i be $P^o_{mi} + jQ^o_{mi}$. In this case, x^o and u^o represent the present operating condition of the system, which includes the fictitious units. The actual operating condition of the system can be obtained by gradually reducing the output of the fictitious generating units to zero and computing the system variables x and u which will make P^o_{mi}, Q^o_{mi} equal to zero. This transition can be achieved along a trajectory which maintains feasibility and optimality. Mathematically, this procedure is formulated as an optimization problem:

Minimize $\quad -\mu\Sigma_i(|dP_{mi}| + |dQ_{mi}|) + \Sigma_j f_j(P_{gj})$

Subject to
Power balance equation
Voltage constraints
Circuit flow constraints
New MW interchange constraints
Unit real and reactive power output constraints
Other pertinent constraints

Finally, the variables dP_{mi} and dQ_{mi} should be constrained with:

$$-P^0_{mi} \le dP_{mi} \le 0 \quad , \quad i = 1,\ldots,n$$
$$-Q^0_{mi} \le dQ_{mi} \le 0 \quad , \quad i = 1,\ldots,n \qquad (1)$$

where $dP_{mi} = P_{mi} - P^o_{mi}$, $dQ_{mi} = Q_{mi} - Q^o_{mi}$, μ is an appropriately selected penalty function, and P_{gj} is the real power output of generating unit j. The first term of the objective function is a penalty function which tends to reduce the fictitious mismatches to zero, thus reaching feasibility. The second term of the objective function is a preselected function to be optimized. In the present case, this term expresses the total generation cost. However, it can be substituted with any other function of interest such as total losses, etc.

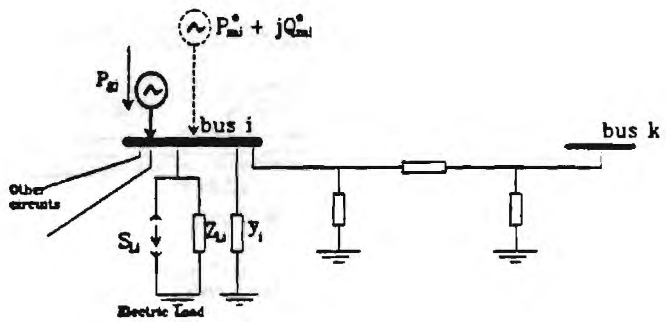

Figure 2. Illustration of a General Bus i of an Electric Power System

The defined optimization problem is a large scale problem. The size of this problem can be drastically reduced with simple transformations. Specifically, observe that the mismatch variables can be substituted with one control variable v as follows:

$$dP_{mi} = -P^{\circ}_{mi}\, v \quad , \quad i = 1,2,\ldots,n$$
$$dQ_{mi} = -Q^{\circ}_{mi}\, v \quad , \quad i = 1,2,\ldots,n \qquad (2)$$

where the variable v represents the normalized change of the mismatch variables ($0 \leq v \leq 1$). Note that this transformation replaces all the mismatch variables (a total of 2n) with a single variable, v.

The variables P_{gj} (a total of m, m is the number of generating units) can be also replaced with only two variables while implicitly incorporating the economic dispatch process. Specifically, consider the economic dispatch problem:

$$\begin{aligned} &\text{minimize} && \sum_j f_j(P_{gj}) = \sum_j \alpha_j + \beta_j P_{gj} + \gamma_j P^2_{gj} \\ &\text{subject to} && \sum_j P_{gj} - q - P_L = 0 \\ & && P^{min}_{gj} \leq P_{gj} \leq P^{max}_{gj} \quad \text{or} \quad P_{gj} = 0 \qquad (3) \end{aligned}$$

where q is the total transmission system losses; P_L is the total system load; $\alpha_j, \beta_j, \gamma_j$ are the quadratic cost coefficients for unit j.

Assume that the present generation schedule is an economic dispatch schedule. This means that P°_{gj}, j = 1,2,...,m is a solution of the above defined optimization problem, satisfying the following equations:

$$\beta_j + 2\gamma_j P^{\circ}_{gj} - \lambda^{\circ}\left(1 - \frac{dq}{dP_{gj}}\right) = 0 \text{ , if } P^{min}_{gj} < P^{\circ}_{gj} < P^{max}_{gj}$$
$$\sum_j P^{\circ}_{gj} - q^{\circ} - P^{\circ}_L = 0 \quad , \quad j = 1,2,\ldots,m$$

λ° is the Lagrange multiplier, or system lamda. Now assume a small change in the variable λ°, $d\lambda$. This change will cause a change in the generating unit outputs and load as follows:

$$dP_{gj} = \begin{cases} \dfrac{1}{2\gamma_j}\left(1 - \dfrac{dq}{dP_{gj}}\right) d\lambda & \text{if } \gamma_j \neq 0 \text{ and } P^{min}_{gj} \leq P^{\circ}_{gj} \leq P^{max}_{gj} \\ 0 & \text{otherwise} \end{cases}$$

$$dP_L = \sum_k \frac{1}{2\gamma_k}\left(1 - \frac{dq}{dP_{gk}}\right)^2 d\lambda \qquad (4)$$

Thus if the electric load increases by dP_L, the unit j should increase by dP_{gj} above. Thus, the economic participation factor, p_{fj}, for unit j is:

$$p_{fj} = \begin{cases} \dfrac{\dfrac{1}{2\gamma_j}\left(1 - \dfrac{dq}{dP_{gj}}\right)}{\sum_k \dfrac{1}{2\gamma_k}\left(1 - \dfrac{dq}{dP_{gk}}\right)^2} & \text{if } \gamma_j > 0 \text{ and } P^{min}_{gj} \leq P^{\circ}_{gj} \leq P^{max}_{gj} \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

In order to allow flexibility in the model, two economic participation factors are defined, one for generation increase and another for generation decrease. Let w_1 be a variable representing total generation increase and w_2 be a variable representing total generation decrease. In this case:

$$dP_{gj} = p^{+}_{fj} w_1 + p^{-}_{fj} w_2 \qquad (6)$$

where:

p^{+}_{fj} is the economic participation factor for generation increase. It is equal to p_{fj} if $P^{min}_{gj} \leq P^{\circ}_{gj} \leq P^{max}_{gj}$ and equals zero if $P^{\circ}_{gj} = P^{max}_{gj}$.

p^{-}_{fj} is the economic participation factor for generation decrease. It is equal to p_{fj} if $P^{min}_{gj} < P^{\circ}_{gj} \leq P^{max}_{gj}$ and equals zero if $P^{\circ}_{gj} = P^{min}_{gj}$.

Above transformation reduces the variables P_{gj}, j = 1,2,...,m to only two variables w_1 and w_2. On the other hand, it guarantees that any changes in unit output according to Equation (6) will preserve an optimal economic dispatch.

Upon substitution of above transformations and linearization of above problem around the present operating point yields:

$$\text{Minimize} \quad -\mu\Sigma\left(|P^{o}_{mi}| + |Q^{o}_{mi}|\right) v + c_1 w_1 + c_2 w_2 + c^T du$$

$$\begin{aligned}
\text{Subject to} \quad & a_1 v + a_2 w_1 + a_3 w_2 + a_4^T du = b \\
& \text{Linearized voltage constraints} \\
& \text{Linearized circuit flow constraints} \\
& \text{Linearized net MW interchange constraints} \\
& \text{Other linearized constraints} \\
& du^{min} \le du \le du^{max} \\
& 0 \le v \le 1.0 \\
& 0 \le w_1 \le w_1^{max} \\
& 0 \le w_2 \le w_2^{max}
\end{aligned} \qquad (7)$$

where u are other available controls such as units off-economic dispatch, transformer taps, switched capacitors and inductors, etc.

The linearized model is valid only in a small region around the operating point [10]. Define:

$$y = \begin{pmatrix} v \\ w_1 \\ w_2 \\ du \end{pmatrix}$$

Then the region of validity may be expressed as follows:

$$y^{min} \le y \le y^{max}$$

The overall problem is stated as follows:

$$\text{Minimize} \quad \left[-\mu\Sigma\left(|P^{o}_{mi}| + |Q^{o}_{mi}|\right), c_1, c_2, c^T\right] * y$$

$$\begin{aligned}
\text{Subject to} \quad & Ay = b \\
& y^{min} \le y \le y^{max}
\end{aligned} \qquad (8)$$

Solution of this problem provides the new generation schedule, new settings of controls, and new power mismatches. A nonzero value of power mismatches indicates that the algorithm has not converged yet. The computed solution is subsequently implemented, the power flow solution is updated, and the process is repeated until the variables dP_{mi}, dQ_{mi} become zero. Note that any generation increase ($w_1 \ne 0$) or decrease ($w_2 \ne 0$) is distributed to the individual generating units according to Equation (6), thus preserving optimality. In case one or more operating constraints are violated, the algorithm ignores the cost oriented objective function and applies a remedial action algorithm [10] to force a feasible operating point. The remedial action method may require that one or more generating units must be removed from the economic dispatch and be controlled individually to maintain feasibility of operating state. This means that the conomic participation factors p^{+}_{fj} and p^{-}_{fj} for this unit or units will be set to zero, and the unit(s) will appear in the controls vector u. When feasibility has been reinstated, the algorithm continues. The overall algorithm is illustrated in Figure 3. The algorithm incorporates procedures for selection of violated constraints to be included in the optimization model (based on coherency of constraints) and model reduction methods which have been introduced in Reference 10.

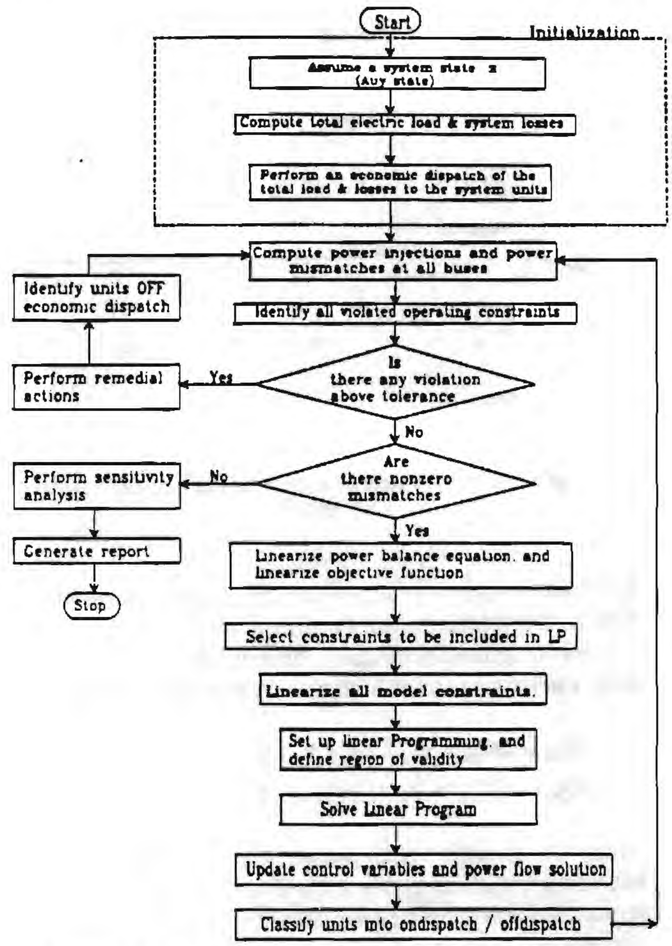

Figure 3. Flow Chart of the Nondivergent/Optimal Power Flow

The proposed approach to the optimal power flow problem has several advantages. First, it combines the remedial actions with the power flow solution and, thus, increases the efficiency of the overall model as compared to performing a power flow solution and then a remedial action separately. Second, it eliminates the necessity of adjustments during the power flow solutions such as net MW export adjustments, capacitor/reactor switching with local logic, etc. These adjustments may require several power flow iterations. Third, it preserves at each iteration an optimal economic dispatch since generation adjustments are performed according to Equation (6) and the variables w_1 and w_2 are computed as to minimize the objective function. (Note that Equation (6) resembles the real time operation of a power system where any changes of the electric load are allocated to the individual units according to their economic participation factors.) Fourth, and most important, it guarantees convergence since the solution always moves towards decreasing mismatches P_{mi} and Q_{mi}.

Efficiency-wise, the proposed power flow/optimization algorithm is competitive with the usual power flow with interchange adjustments, capacitor/reactor switching, etc. The efficiency of the method is quantified later.

5. MODEL LINEARIZATION

As is illustrated in Figure 3, each major iteration of the algorithm requires a model linearization and subsequent solution via a linear program. The linearization involves three distinct tasks: (1) linearization of the objective function, (2) linearization of the power balance equation, and (3) possible linearization of operating constraints which must be included in the model. The selection of constraints to be included in the model uses the method developed in Reference 10. Specifically, a coherency analysis of all violated constraints is performed to determine groups of coherent constraints and subsequent selection of one leading constraint from each coherent group. This process minimizes the number of constraints to be included in the model. The linearization of the constraints included in the model is based on the costate method described in Reference 10. The linearization of the objective function and power balance equation uses the same method. It will be shown that the linearization of the objective function and the power balance equation can be performed with only one costate vector solution, i.e. only one forward and back substitution. The linearization of the operating constraints requires one forward and back substitution per constraint, as is described in [10].

5.1 Linearization of the Objective and Power Balance Equation

For this purpose, the linearized expression for the transmission losses is computed first. The general expression of the sensitivity of the transmission losses **q** with respect to a control variable is:

$$\frac{dq}{du} = \frac{\delta q}{\delta u} - \hat{x}^T \frac{\delta g}{\delta u} \tag{9}$$

$$J^T \hat{x} = \frac{\delta q}{\delta x} \tag{10}$$

where:

- δ represents partial derivative
- u is the control variable of interest
- g represents the power flow equations
- $\hat{x}$ represents the costate vector solution
- J is the Jacobian matrix.

Above computation provides the derivatives dq/dP_{gi} and thus the penalty factors $(1 - dq/dP_{gi})$. Subsequently, using Eqs. (5) and (6), the economic participation factors p^+_{fj} and p^-_{fj} are computed. The linearized objective function is:

$$\sum_j f_j(P^o_{gj}) + w_1 \sum_j \left(\beta_j + 2\gamma_j P^o_{gj}\right) p^+_{fj} + w_2 \sum_j \left(\beta_j + 2\gamma_j P^o_{gj}\right) p^-_{fj} = \sum_j f_j(P^o_{gj}) + c_1 w_1 + c_2 w_2 \tag{11}$$

Combining all these results, the linearized power balance equation is:

$$a_1 v + a_2 w_2 + a_3 w_2 + a_4^T u = b \tag{12}$$

where:

$$a_1 = \sum_i \left(\hat{x}_{pi} P^o_{mi} + \hat{x}_{qi} Q_{mi}\right)$$

$$a_2 = \sum_j \hat{x}_{uj} p^+_{fj}$$

$$a_3 = \sum_j \hat{x}_{uj} p^-_{fj}$$

In above computational procedure, the major task is the solution of Equation (10) which is equivalent to one forward and back substitution using the triangular factors of the Jacobian matrix.

6. METHOD EVALUATION

The proposed method has been tested with several power systems. The results of testing with two systems will be presented: (1) the IEEE Reliability Test System (24 buses), and (2) the Georgia Power Company bulk power system (1304 buses, 1546 circuits, 117 transformers [33 variable taps], 81 capacitor banks, 137 generating units).

Convergence performance for the IEEE 24 bus Reliability Test System are illustrated in Table 1. The table lists for each major iteration: (1) type of iteration, OPF means optimal power flow iteration and RA means remedial action iteration; (2) the total number of constraints and the number of constraints included in the LP model, (3) the maximum real and reactive power mismatch; (4) the total generation cost; and (5) the total transmission loss. This table clearly illustrates the essence of the algorithm. It starts from an optimal solution with practically unloaded network, minimal losses, and minimal total generation cost. As the mismatches decrease, the network is loaded, the losses increase, and the total generation cost increases. At the solution, the mismatches go to zero, the total losses are 98.78 MW and the generation cost is 50289.72 \$/hr. Note that the algorithm had to switch to the remedial actions algorithm twice because of encountered infeasibility (violated operating constraints). It is interesting to compare this solution to the usual power flow solution. The usual power flow solution yields:

Total Generation Cost:	53380.664 \$/hr
Total Transmission Losses:	56.654 MW

Note that losses are lower but cost is much higher.

Convergence performance for the Georgia Power Company 1304 bus system are illustrated in Table 2. The same format as Table 1 is used. Note that the algorithm switches to remedial actions twice for this system. The optimal solution is:

Total Generation Cost:	335767.65 \$/hr
Total Transmission Loss:	475.89 MW

The usual power flow solution with a generation dispatch specified by Georgia Power Company is:

Table 1. Convergence Performance of the Proposed Method 24 Bus RTS System

Iteration #/Type	Constraints Total/in LP	Max Power Mismatch Real/Reactive (in pu)	Total Generation Cost ($/hr)	Total System Loss(pu)
1 / OPF	1 / 1	8.0000 / 1.4595	45148.83	0.0337
2 / OPF	1 / 1	6.7346 / 1.2068	45285.06	0.1334
3 / OPF	1 / 1	5.4717 / 0.9597	45613.51	0.2993
4 / OPF	1 / 1	4.2109 / 0.7183	46149.63	0.5339
5 / OPF	1 / 1	2.9517 / 0.4837	46927.08	0.8435
6 / OPF	1 / 1	1.6936 / 0.2579	48027.92	1.2423
7 / RA	7 / 5	1.6936 / 0.2579	48188.80	1.1027
8 / RA	8 / 6	0.6936 / 0.1579	48356.33	1.0580
9 / OPF	8 / 6	0.0952 / 0.0642	49844.01	0.9928
10 / OPF	8 / 6	0.0083 / 0.0100	50289.72	0.9878

Table 2. Convergence Performance of the Proposed Method 1304 Bus System

Iteration #/Type	Constraints Total/in LP	Max Power Mismatch Real/Reactive (in pu)	Total Generation Cost ($/hr)	Total System Loss(pu)
1 / OPF	1 / 1	33.4200 / 3.7840	319430.87	0.4488
2 / OPF	1 / 1	23.3997 / 2.7741	322634.50	1.8802
3 / RA	10 / 3	23.3997 / 2.7741	322634.50	1.8862
4 / OPF	10 / 3	13.3449 / 1.5822	329629.50	3.9068
5 / OPF	10 / 3	3.3713 / 0.5959	335517.90	4.8088
6 / OPF	10 / 3	0.0842 / 0.1029	335597.75	4.7560
7 / RA	11 / 4	0.0129 / 0.0155	335767.65	4.7998
8 / OPF	11 / 4	0.0074 / 0.0096	335767.65	4.7589

Table 3. Performance Evaluation of the Proposed Method 1304 Bus System/Times Are in Seconds on a PS/2, Model 70, 25 Mhz PC

Iteration #/Type	Constraints Total/in LP	Model Linearztn	LP Setup and Solution	Power Flow Update	Total Execution Time
1 / OPF	1 / 1	0.87	0.93	5.43	7.63
2 / OPF	1 / 1	0.54	2.30	5.54	8.68
3 / RA	10 / 3	1.64	0.82	5.43	8.68
4 / OPF	10 / 3	1.12	3.62	5.43	10.13
5 / OPF	10 / 3	1.09	3.90	5.49	10.76
6 / OPF	10 / 3	1.09	3.57	5.49	10.43
7 / RA	11 / 4	1.64	4.94	5.49	12.80
8 / OPF	11 / 4	2.08	2.91	5.49	10.76

Total Generation Cost: 400460.625 $/hr
Total Transmission Losses: 406.39 MW

Performance evaluation is given in Table 3 for the Georgia Power Company system (1304 bus system). The table provides for each iteration the number of total constraints and the number of constraints included in the LP model, and the execution times of the three major components of the algorithm, i.e. model linearization, LP setup and solution, and power flow update. The power flow update timing includes the recomputation and refactorization of the Jacobian matrix. The table also includes the total execution time for each major iteration. It should be clear that the total execution time is the sum of the numbers in the last column of Table 3. Note that the total execution time of the nondivergent/optimal power flow is 79.87 seconds on a PS/2, Model 70, 25 MHz PC. On the same computer, the usual power flow solution using a reasonably optimized fast decoupled power flow algorithm requires 46 seconds. Thus, the proposed nondivergent/optimal power flow exhibits competitive execution times with the usual power flow.

One very important observation related to the performance of the method is that violated operating constraints appear gradually because the network is gradually loaded. Appearance of an overwhelming number of violated constraints, as can happen at the end of a conventional power flow solution, is improbable with the proposed method.

8. CONCLUSIONS

A unified framework for the formulation and solution of the nondivergent and optimal power flow is proposed. The method introduces fictitious generators at each bus which represent the power mismatches. The output of the fictitious generators is reduced while the system state is directed in a trajectory which maintains feasibility and optimality. The process guarantees convergence, if a solution exists, and optimality with respect to a specified objective function. The efficiency of the method is competitive with the usual power flow algorithms.

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An Analytic Method for Composite Power System Simulation

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Abstract

An analytic simulation method of the composite power system analysis is proposed which takes into consideration the uncertainty of the electric loads, availability of generating units and transmission lines, nonlinearities in the power flow equations, as well as the major operating practices such as economic dispatch. The method is based on the following procedure. First, given the probabilistic electric model, the probability distribution function of power injections at generation buses is computed as a function of a small number of independent random variables by simulating generating unit forced outages and economic dispatch practices. Next, circuit flows, bus voltage magnitudes, etc are expressed as linear combinations of power injections at generation buses. This relation allows the computation of the distribution functions of circuit flows, bus voltage, transmission losses, etc. The method has been validated by comparing it to the exact results for a three bus system calculated by complete enumeration.

1. Introduction

Probabilistic methods have been long ago recognized to be the premier tool for power system expansion planning and to a lesser degree for operation planning. In the presence of increased deregulation and competition, uncertainty increases and probabilistic methods become a must. In the past, probabilistic methods were applied separately to the generation and transmission system. Past design practices of a power system and its operating practices justified this decomposition. Presently however, and in the future, this decomposition is not justified. Recent trends have resulted in transmission constrained systems which means that there is substantial impact on reliability, security, and cost due to the interaction between generation and transmission systems. The interaction cannot be ignored anymore, thus the need for composite power system simulation methods.

Efforts to develop composite power system simulation methods date at least 15 years ago. Three distinct approaches are identified in composite power system simulation:

1. Monte Carlo simulation
2. The enumerative approach
3. Analytic simulation

Brief description of these approaches follows.

Monte Carlo Simulation Monte Carlo simulation is easily implementable and provides a great tool for valuation of other methods. It is imperative that the Monte Carlo simulation be based on comprehensive models of electric load, generation, transmission, etc. The level of modeling detail can vary depending on the objective of the study. In Monte Carlo method, the number of trials must be large for meaningful results. This requirement hinders the applicability of this method to large scale power systems. The Monte Carlo simulation is typically limited to small or medium size power systems. Recent efforts focus on decreasing the computational burden by application of variance reduction methods, importance sampling, etc.

The Enumerative Approach The enumerative approach has been extensively used in North America for adequacy evaluation/reliability analysis. The basic idea is to enumerate all possible states of an electric power system, to analyze the states and store the results for subsequent processing. The conceptual and computational problems are serious. As an example, a very large number of multiple generating unit outages must be considered since the probability of these events is substantial (generating unit probability of unavailability is quit high). The problem of a priori determining the severity of multiple unit outages is also a challenging problem. For practicality, efforts related to this method focus on identification of contingency states which substantially affect reliability or identification of minimal cut states.

Analytic Simulation The analytic simulation method also known as the probabilistic power flow was first developed in the early 1970's by Borkowska, Allen et al. The first analytic model was simple using the DC network model and the assumption that the electric loads are independent random variables. This model is unrealistic in the sense that it does not reflect the actual power system. Since then, several analytic methods were proposed which try to simulate the actual power system as close as possible. The major issues associated with the analytic methods are following:

a) Correlation between the nodal power injections.
b) Nonlinearities of the power system model.
c) Effects of operating practices such as the economic dispatch.
d) Uncertainty associated with the availability of generation units and transmission lines.

This paper describes an improved analytic method for the composite power system analysis. The method addresses the above four important issues. Validation of the proposed method is performed by a simplified three bus system.

2. System Model Description

The analytic method is based on comprehensive models of (a) electric load, (b) generating system and (c) transmission system. Description of these models follows:

2.1 The Probabilistic Electric Load Model.

The electric model is illustrated in Figure 1. It is a multiple input/multiple output ARIMA model. Specifically, the bus electric load, represented with the vector $S_L(t)$, is constructed from a vector of m independent white noise processes $\eta(t)$. Through an ARIMA model, the independent white noise processes are converted into a vector of stationary stochastic processes z(t). The vector z(t) is inverted to provide a vector of nonstationary stochastic processes x(t). Finally, the vector x(t) is translated into bus electric loads with the linear system L. The described model is represented with the following equations:

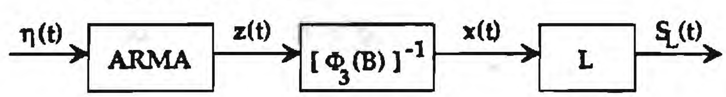
$$\Phi_1(B)z(t) = \Phi_2(B)\eta(t) \quad (1)$$

$$\Phi_3(B)z(t) = x(t) \quad (2)$$

$$S_L(t) = Lx(t) \quad (3)$$

where

$\eta(t)$	is an m-vector of independent white noise processes
z(t)	is an m-vector of stationary stochastic processes
x(t)	is an m-vector of nonstationary stochastic processes
$S_L(t)$	is an n-vector of bus electric loads
$\Phi_1(\cdot),\Phi_2(\cdot),\Phi_3(\cdot)$	are vectors of arbitrary polynomials of the argument
B	is the backward operator
L	is an nxm matrix



(a)

$S_L = P_o + P v$

P an nx1 vector

Po an nxm matrix

v an mx1 random variable

(b)

Figure 1. Proposed Electric Load Model
(a) Electric Load Represented As Stochastic Process
(b) Electric Load Represented As Random Variable

ARIMA models have been extensively used to represent the electric load. It is well known that they are capable of representing the periodicities as well as the nonstationary property of electric load. The innovation introduced here is the linear model L which translates the low order nonstationary stochastic process vector x(t) into the high order vector $S_L(t)$ of the bus electric loads. This innovation is justified on the basis that bus electric loads are typically strongly correlated. It is, therefore, reasonable to assume that they are generated as a linear combination of a small number of independent stochastic processes.

The optimal order of the ARIMA model (order of functions, Φ_1,Φ_2 and Φ_3) and the number of independent white noise processes (vector $\eta(t)$) is system dependent. Historical data of hourly bus electric loads can be utilized to identify the optimal order and the parameters of the described electric load model. The equations (1), (2) and (3) can be combined to yield:

$$S_L(t) = L\,\Phi_3(B)\,\Phi_1^{-1}(B)\,\Phi 2(B)\,\eta(t) \tag{4}$$

It is well known that above equation can be approximated with a finite order polynomial of the backward operator B (moving average model). This approximation yields

$$S_L(t) = P_0 + \sum_{i=0}^{N} L_i \eta(t-i) \tag{5}$$

A further simplification is to consider the electric load at a specified time interval. In this case the stochastic processes $S_L(t)$ and $\eta(t)$ become random variables (independent of time). Equation (5) then becomes

$$S_L = P_0 + P_1 v \tag{6}$$

where

S_L	n-vector of random variables representing the bus electric loads
v	m-vector of random variables
P_0,P_1	appropriately dimensioned vector and matrix.

2.2 The Generating System Model.

Each generating unit is represented with a 2-state Markov model. State 1 is the UP state and state 2 is the DOWN state. The probability of state 1 for unit i is p_i. In addition, each unit is characterized with an operating production cost expressed in terms of a quadratic function

$$f_i(P_{gi}) = a_i + b_i P_{gi} + C_i P_{gi}^2 \tag{7}$$

At every instant of time, the generation system should be operated in such a way as to minimized the production cost. This is expressed in terms of the economic dispatch problem:

$$\begin{aligned} &\text{Minimized} \quad \sum_{i=1}^{m} (a_i + b_i P_{gi} + c_i P_{gi}^2) \\ &\text{Subject to} \quad \sum_{i=1}^{m} P_{gi} - P_{loss} - P_{load} = 0 \\ &\qquad\qquad\quad 0 \le P_{gi} \le P_{gi,max} \end{aligned} \tag{8}$$

2.3 The Transmission System Model.

The proposed transmission model comprises a collection of transmission units. A transmission unit is defined as a set of transmission equipment (transmission lines, transformers, breakers, etc.) with correlated failure rates. For simplicity a transmission unit is represented with a 2-state Markov model. This model is very general and capable of representing single circuit outages, as well as common mode outages, station equipment failures, and protection system failures. As for units, state 1 is the UP state and state 2 is the DOWN state. The probability of state 1 for transmission unit i is p_i.

3. Analytic Simulation Method Description

The proposed method provides probabilistic characterizations of circuit flow and bus voltage magnitudes for the given electric load and generation system model. Specifically, the probability distribution function of circuit flow S_l and the bus voltage magnitude V_i for each circuit l and bus i is computed. This is achieved with a two step procedure. In the first step, the electric load and generating system model is used to characterize the power injections, Y, at the system buses as random variables. This is illustrated in Figure 2b. The random variables, Y, are in general correlated. Subsequently, a probabilistic power flow provides the probability distribution function of S_l, V_i from the probabilistic model of the injection Y. Details are provided next.

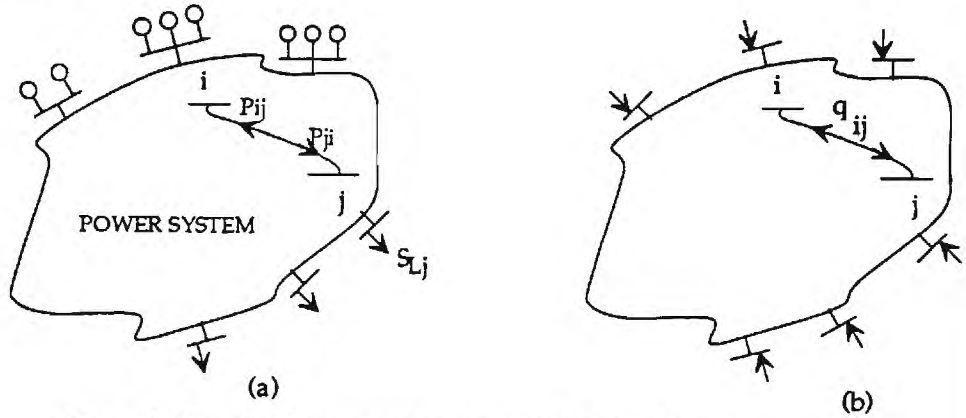


Figure 2. Schematic Representation of an Electric Power System
(a) Electric Power System Comprising Generation, Transmission and Loads
(b) Replacement of Generation and Loads with Injection Variables

3.1 Generation System Simulation Method

The objective of the generation system simulation method is to provide a probabilistic description of the bus power injections by taking into consideration the major operational practices and constraints of the system. Example of operating practices are: (a) economic dispatch, (b) dispatch of hydro units, (c) interchange schedules, (d) remedial actions, (e) security dispatch, (f) other. Examples of constraints are: (a) unit forced outages, (b) unit maintenance, (c) available hydro energy, (d) allowable range of bus voltages, (e) allowable loading level of circuits, etc. In subsequent paragraphs, the proposed simulation method will be outlined and an explanation will be given of how it accounts for operating practices and constraints.

Consider the proposed stochastic load model and the Markov model of generating unit availability. Further, consider the operation of the system during a specified period of time. This period of time may be contiguous or noncontiguous (for example, 1 pm to 3 pm each day for a period of one year). Utilizing the proposed electric load model, the electric load of a bus, during the period of simulation, can be characterized as a random variable with a probability distribution function and correlation to other bus loads. The total electric load can be characterized as a random variable, L, with a probability distribution function $F_L(l)$. Similarly, utilizing the Markov model of generating units, the unit availability can be represented as a discrete random variable. Assume that the n units of the system operate at level $x_1, x_2, ..., x_n$, respectively, while the total electric load is l. If unit k is not in operation, then obviously x_k equals 0. Since there is a finite probability that any unit can be forced out, the output of unit j, x_j, can be considered to be a random variable with probability of force outage equal to q_j. We write

$$\Pr(A_j = x_j) = 1 - q_j,\ x_j \neq 0 \tag{9}$$
$$\Pr(A_j = 0) = q_j \tag{10}$$

Where A_j is a random variable representing the available capacity of unit j, q_j is the probability of unavailability. The above relationships state that the probability that the output of generator j is x_j equals $1 - q_j$, and the probability that the same output is zero equals q_j.

For the condition that has been considered, the apparent load ℓ_a will be

$$\ell_a = \ell - x_1 - x_2 - \dots - x_n \tag{11}$$

Since ℓ, x_1, x_2, ... x_n, are not deterministically known, the above equation can be replaced with its equivalent equation in term of the corresponding random variables

$$L_a = L - A_1 - A_2 - \dots - A_n \tag{12}$$

Where L is a random variable representing the local electric load and A_i is a random variable representing the output of unit i. Since the probability distributions of the random variable L, A_1, A_2, ..., A_n are known and since these random variables are independent, the probability distribution function of the random variable L_a is easily computed with a series of convolutions.

If we assume that $\ell_a > 0$ (that is, load exceeds generation), then another unit should be brought into operation or one or more of the operating units should increase their output. Assume that the unit i is operating at x_i and that it is selected according to a criterion to respond to any increases in the load. We shall refer to this criterion as the dispatch criterion. It is defined as to satisfy operational practices and constraints. The dispatch criterion will be qualified later. Without loss of generality, x_i may be equal to zero. In general, if $\ell_a > 0$, the output of unit i will increase from x_i to $x_i + \Delta x_i$, where Δx_i is a small increment (1-2 MW). We shall refer to this increment as the block Δx_i. The described formulation and direct application of probability theory, yields expressions for the expected energy, cost of operation, required fuel, etc., from the Δx_i increase in the output of generator i. The detailed mathematical formulation is given in [1].

Upon completion of the simulation algorithm, the probability that unit i operates at level x_i or the probability density function of power injections at the generation buses as well as the joint probability density of any generating unit pair has been constructed. Thus, the power injections to the electric power network are characterized as random variables with known probability distribution functions and correlations. It should be emphasized that the probability distribution function at generation buses can not be approximated with Guassian distributions. This basic result can also provide the performance parameters for each generating unit, such as: (a) expected time of operation, (b) expected produced energy, (c) expected production cost, etc. [1].

The proposed method is also capable of simulating the operating practices and constrains of a power system. This objective is achieved by appropriate selection of the dispatch criterion mentioned in the description of the method. Specifically, the dispatch criterion is defined as the sum of the actual operating cost of the generating unit plus a nonlinear penalty function of generating unit output defined with m parameters. In this way any arbitrary dispatch criterion can be accomodated. the parameters of the penalty are selected with a probabilistic optimization method which is described later. The solution of the optimization problem provides the parameters of the penalty function, such as the operational practices and constraints will be satisfied with maximum probability. Note that if the nonlinear penalty function is neglected, then the dispatch criterion equals the actual operating cost of the generating unit. This selection amounts to simulating the economic dispatch only, neglecting other operating practices, such as interchange schedules, etc., and operating constraints, such as hydro energy limitations, transmission limitations, etc.

3.2 Transmission System Simulation Method

The simulation method of a transmission system consists of transforming the probability density function of bus power injections into the probability density function of system output variables such as circuit loading, bus voltages, and system losses. This transformation can be achieved with any desirable degree of accuracy. Subsequently, the probability density functions of system output variables can be easily translated into reliability indices, total system losses, etc.

For the purpose of computing the probability distribution functions of output variables, an extension of the method reported in [3] will be employed. Specifically, consider an output variable v_{oi}. The output variable v_{oi} may be a bus voltage, the power flow on a circuit, the total system losses, the net interchange, etc. This variable is expressed as a function of the power injections:

$$v_{oi} = \bar{v}_{oi} + \sum_j f_{oij}(x_j - \bar{x}_j) \tag{13}$$

where

$\bar{x}_{ji}$ is the expected value of the power injection at bus j (x may represent real or reactive power)

x_j is the power injection at bus j

$f_{oij}(\cdot)$ is, in general, a nonlinear function of the argument.

For practical calculations, the function $f_{oij}(\cdot)$ is expressed as a piecewise linear function:

$$f_{oij}(x_j - \bar{x}_j) = \{ s^k_{oij}\, y;\ x_j^{k-1} - \bar{x}_j \le y \le x_j^k - \bar{x}_j;\ k=1,2,\dots \} \tag{14}$$

where

$$s^k_{oij} = dv_{oi} / dx_j \qquad \text{computed at } y = (x_j^{k-1} + x_j^k - 2\bar{x}_j)/2$$

References [3] and [6] provide an efficient procedure for computing the sensitivities s^k_{oij} based on the costate (adjoint) equation.

Equation (14) states that the function $f_{oij}(\cdot)$ can be considered linear in an interval $[x_j^{k-1} - \bar{x}_j, x_j^k - \bar{x}_j]$. The interval of validity of the linearized model depends on the parameters of the system in the neighborhood of bus j. A simple method for selecting the bounds $(x_j^{k-1} - \bar{x}_j)$ and $(x_j^k - \bar{x}_j)$ has been developed and reported in [6]. The method is based on the observation that the error is approximately an invariant function of the quantities $(P_{inj}/\Sigma Y)$ and $(Q_{inj}/\Sigma Y)$. where P_{inj} and Q_{inj} are real and reactive power injections at a bus and ΣY is the sum of admittances connected to the bus. For a given allowable error, these quantities must not exceed the threshold values ε_1 and ε_2 yielding the bounds $(x_j^{k-1} - \bar{x}_j)$ and $(x_j^k - \bar{x}_j)$. It is important to note that the interval of validity of the linearized model is quite wide even for small allowable threshold values, i.e., 2%. Practically, this means that variations of bus electric loads can be accurately represented with only one segment of the piecewise linear equation (14). More than one segment is needed only at generation buses, or buses with very large load, where the power injection variations are high.

Within the simulation method, the effects of circuit outages can be accounted for. For this purpose, the nonlinear effects of circuit outages are be accounted with a two step procedure: In the first step, the most important circuit outages are be identified with a ranking algorithm

[4]. In the second step, a probabilistic load flow computation is performed for a set of highly ranked circuit outages.

4. Method Validation

4.1 Example Test System

A three bus system has been used as the example test system to validate the proposed analytic simulation method for the composite power system. The system network is illustrated in the Figure 3. The system data is described following.

a) Electric Load Model

The electric load model of the test system is

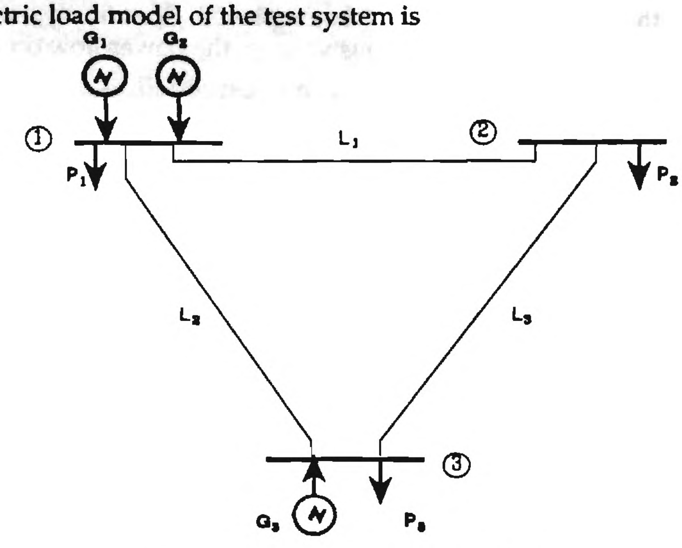


Figure 3. Example Three Bus System

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 80 \\ 120 \end{bmatrix} + \begin{bmatrix} 25 & 20 \\ 15 & 18 \\ 20 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Where v_1, v_2 are independent Guassian variables with

$E[v_i] = 0$

$Var(v_i) = 1.0$

b) The Generating System model:

Unit No.	Pg,max (MW)	Cost Coefficient a	b	c	Availability
1	200	0	20	0.02	0.90
2	300	0	0	0	1.0
3	150	0	22	0.02	0.95

c) The Transmission System Model:

For purposes of validating the proposed method, we use a simplified DC network model. In this model the real power flow through the circuit can be represented as:

$$P_{lm} = \gamma_{lm} (\delta_l - \delta_m)$$

where

P_{lm} is real power flow from terminal l to terminal m

δ_l is the voltage phase angle at terminal l

δ_m is the voltage phase angle at terminal m

η_{lm} is the transmission unit constant which is approximately equal to the total admittance of the transmission unit

The example test system has the following transmission system model.

Line L_1 : $p_1 = 1.0$ $\gamma_1 = 15$

Line L_2 : $p_2 = 1.0$ $\gamma_2 = 12$

Line L_3 : $p_3 = 0.97$ $\gamma_3 = 8$

4.2 Sample Results

With the given electric load model, the total electric load is represented with

$$L = 250 + 60v_1 + 43v_2$$

Then the generating unit outputs are represented as a linear combination of the independent random variables v1 and v2 by taking into consideration the economic dispatch effect. Subsequently, generation bus power injections are easily expressed from the generating unit outputs. For the simplified three bus system, the result can be expressed with analytical functions. For example the output of unit 1 can be expressed as an analytical function of random variable v_1 and v_2 with respect to the economic dispatch.

$$P_{G1} = \begin{cases} 0 & 60v_1 + 43v_2 \leq 50 \\ 60v_1 + 43v_2 - 50 & 50 < 60v_1 + 43v_2 \leq 100 \\ 30v_1 + 21.5v_2 & 60v_1 + 43v_2 > 100 \end{cases}$$

Since v_1 and v_2 are Gaussain distributed, the distribution function of unit 1 can be represented by the standard Gaussian distribution.

$$F_{G1}(x) = \Pr\{P_{G1} \leq x\} = \begin{cases} 0 & x < 0 \\ \Phi\left(\frac{x+50}{73.82}\right) & 0 \leq x < 50 \\ \Phi\left(\frac{x}{36.91}\right) & x \geq 50 \end{cases}$$

where: $\Phi(\cdot)$ is the standard Guassian distribution.

Similarly, probability distribution functions can be exactly computed for other quantities. On the other hand the same probability distribution functions can be computed with the proposed analytic method and subsequently compared to the exact result. Examples of this comparison follow. Figure 4 shows the probability distribution of the power injection at bus 1 computed with the proposed method. The computed distribution using exact equations for this simple example are superimposed with dashed lines. Figure 5 illustrates the circuit power flow distribution. Again the results of the simulation method are superimposed on the exact results. It is obviously the proposed method provides has the good agreement with exactly computed results.

5. Conclusions

An analytic method for composite power system simulation has been described. A simple example was utilized to validate the method. For the simple example, the exact probability distribution functions have been computed and compared to results of the method. The comparison is favorable.

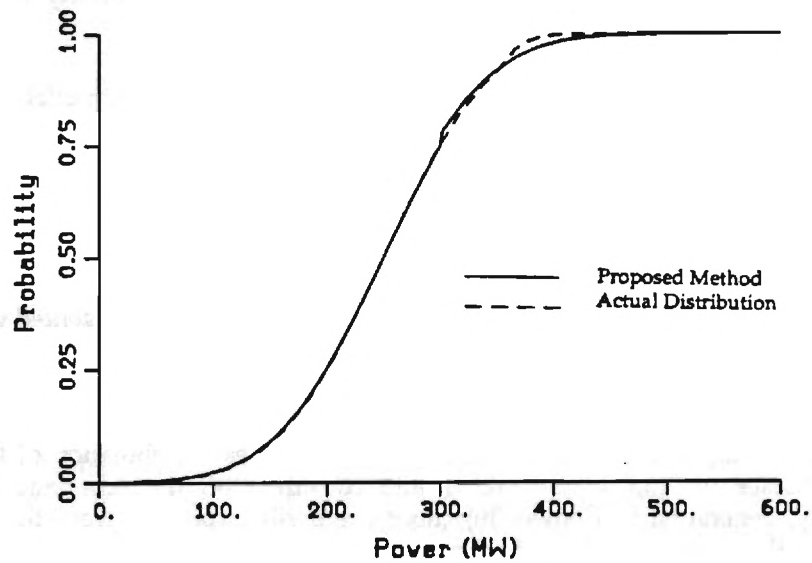


Figure 4. Probability Distribution Function of the Power injection at Bus1

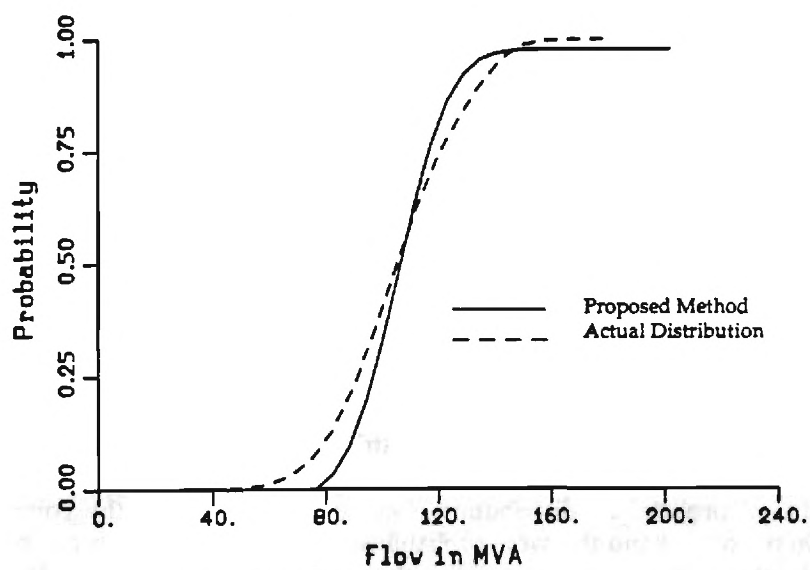


Figure 5. Probability Distribution Function of the Circuit 1 - 3 Power Flow

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Biosketches

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APPENDIX J: An Analytic Method for Composite Power System Simulation

A. P. Sakis Meliopoulos and Feng Xia, "An Analytic Method for Composite Power System Simulation" Submitted to the 29th North American Power Symposium, Carbondale, Illinois, October 7-8, 1991.

An Analytic Method for Composite Power System Simulation

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Abstract

An analytic simulation method of the composite power system analysis is proposed which takes into consideration the uncertainty of the electric loads, availability of generating units and transmission lines, nonlinearities in the power flow equations, as well as the major operating practices such as economic dispatch. The method is based on the following procedure. First, given the probabilistic electric model, the probability distribution function of power injections at generation buses is computed as a function of a small number of independent random variables by simulating generating unit forced outages and economic dispatch practices. Next, circuit flows, bus voltage magnitudes, etc are expressed as linear combinations of power injections at generation buses. This relation allows the computation of the distribution functions of circuit flows, bus voltage, transmission losses, etc. The method has been validated by comparing it to the exact results for a three bus system calculated by complete enumeration.

1. Introduction

Probabilistic methods have been long ago recognized to be the premier tool for power system expansion planning and to a lesser degree for operation planning. In the presence of increased deregulation and competition, uncertainty increases and probabilistic methods become a must. In the past, probabilistic methods were applied separately to the generation and transmission system. Past design practices of a power system and its operating practices justified this decomposition. Presently however, and in the future, this decomposition is not justified. Recent trends have resulted in transmission constrained systems which means that there is substantial impact on reliability, security, and cost due to the interaction between generation and transmission systems. The interaction cannot be ignored anymore, thus the need for composite power system simulation methods.

Efforts to develop composite power system simulation methods date at least 15 years ago. Three distinct approaches are identified in composite power system simulation:

1. Monte Carlo simulation
2. The enumerative approach
3. Analytic simulation

Brief description of these approaches follows.

Monte Carlo Simulation Monte Carlo simulation is easily implementable and provides a great tool for valuation of other methods. It is imperative that the Monte Carlo simulation be based on comprehensive models of electric load, generation, transmission, etc. The level of modeling detail can vary depending on the objective of the study. In Monte Carlo method, the number of trials must be large for meaningful results. This requirement hinders the applicability of this method to large scale power systems. The Monte Carlo simulation is typically limited to small or medium size power systems. Recent efforts focus on decreasing the computational burden by application of variance reduction methods, importance sampling, etc.

The Enumerative Approach The enumerative approach has been extensively used in North America for adequacy evaluation/reliability analysis. The basic idea is to enumerate all possible states of an electric power system, to analyze the states and store the results for subsequent processing. The conceptual and computational problems are serious. As an example, a very large number of multiple generating unit outages must be considered since the probability of these events is substantial (generating unit probability of unavailability is quit high). The problem of a priori determining the severity of multiple unit outages is also a challenging problem. For practicality, efforts related to this method focus on identification of contingency states which substantially affect reliability or identification of minimal cut states.

Analytic Simulation The analytic simulation method also known as the probabilistic power flow was first developed in the early 1970's by Borkowska, Allen et al. The first analytic model was simple using the DC network model and the assumption that the electric loads are independent random variables. This model is unrealistic in the sense that it does not reflect the actual power system. Since then, several analytic methods were proposed which try to simulate the actual power system as close as possible. The major issues associated with the analytic methods are following:

a) Correlation between the nodal power injections.
b) Nonlinearities of the power system model.
c) Effects of operating practices such as the economic dispatch.
d) Uncertainty associated with the availability of generation units and transmission lines.

This paper describes an improved analytic method for the composite power system analysis. The method addresses the above four important issues. Validation of the proposed method is performed by a simplified three bus system.

2. System Model Description

The analytic method is based on comprehensive models of (a) electric load, (b) generating system and (c) transmission system. Description of these models follows:

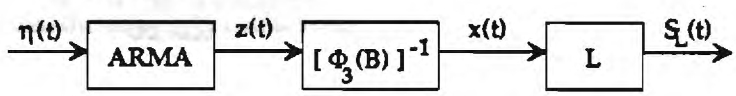
2.1 The Probabilistic Electric Load Model.

The electric model is illustrated in Figure 1. It is a multiple input/multiple output ARIMA model. Specifically, the bus electric load, represented with the vector $S_L(t)$, is constructed from a vector of m independent white noise processes $\eta(t)$. Through an ARIMA model, the independent white noise processes are converted into a vector of stationary stochastic processes z(t). The vector z(t) is inverted to provide a vector of nonstationary stochastic processes x(t). Finally, the vector x(t) is translated into bus electric loads with the linear system L. The described model is represented with the following equations:

$$\Phi_1(B)z(t) = \Phi_2(B)\eta(t) \tag{1}$$
$$\Phi_3(B)z(t) = x(t) \tag{2}$$
$$S_L(t) = Lx(t) \tag{3}$$

where

$\eta(t)$	is an m-vector of independent white noise processes
z(t)	is an m-vector of stationary stochastic processes
x(t)	is an m-vector of nonstationary stochastic processes
$S_L(t)$	is an n-vector of bus electric loads
$\Phi_1(\cdot),\Phi_2(\cdot),\Phi_3(\cdot)$	are vectors of arbitrary polynomials of the argument
B	is the backward operator
L	is an nxm matrix



(a)

$S_L = P_o + P v$

P an nx1 vector
Po an nxm matrix
v an mx1 random variable

(b)

Figure 1. Proposed Electric Load Model
(a) Electric Load Represented As Stochastic Process
(b) Electric Load Represented As Random Variable

ARIMA models have been extensively used to represent the electric load. It is well known that they are capable of representing the periodicities as well as the nonstationary property of electric load. The innovation introduced here is the linear model L which translates the low order nonstationary stochastic process vector x(t) into the high order vector $S_L(t)$ of the bus electric loads. This innovation is justified on the basis that bus electric loads are typically strongly correlated. It is, therefore, reasonable to assume that they are generated as a linear combination of a small number of independent stochastic processes.

The optimal order of the ARIMA model (order of functions, Φ_1,Φ_2 and Φ_3) and the number of independent white noise processes (vector $\eta(t)$) is system dependent. Historical data of hourly bus electric loads can be utilized to identify the optimal order and the parameters of the described electric load model. The equations (1), (2) and (3) can be combined to yield:

$$S_L(t) = L\, \Phi_3(B)\, \Phi_1^{-1}(B)\, \Phi 2(B)\, \eta(t) \tag{4}$$

It is well known that above equation can be approximated with a finite order polynomial of the backward operator B (moving average model). This approximation yields

$$S_L(t) = P_0 + \sum_{i=0}^{N} L_i \eta(t-i) \tag{5}$$

A further simplification is to consider the electric load at a specified time interval. In this case the stochastic processes $S_L(t)$ and $\eta(t)$ become random variables (independent of time). Equation (5) then becomes

$$S_L = P_0 + P_1 v \tag{6}$$

where

S_L n-vector of random variables representing the bus electric loads

v m-vector of random variables

P_0,P_1 appropriately dimensioned vector and matrix.

2.2 <u>The Generating System Model</u>.

Each generating unit is represented with a 2-state Markov model. State 1 is the UP state and state 2 is the DOWN state. The probability of state 1 for unit i is p_i. In addition, each unit is characterized with an operating production cost expressed in terms of a quadratic function

$$f_i(P_{gi}) = a_i + b_i P_{gi} + C_i P_{gi}^2 \tag{7}$$

At every instant of time, the generation system should be operated in such a way as to minimized the production cost. This is expressed in terms of the economic dispatch problem:

$$\begin{aligned} &\text{Minimized} \quad \sum_{i=1}^{m} (a_i + b_i P_{gi} + c_i P_{gi}^2) \\ &\text{Subject to} \quad \sum_{i=1}^{m} P_{gi} - P_{loss} - P_{load} = 0 \\ &\qquad\qquad\quad 0 \le P_{gi} \le P_{gi,max} \end{aligned} \tag{8}$$

2.3 <u>The Transmission System Model</u>.

The proposed transmission model comprises a collection of transmission units. A transmission unit is defined as a set of transmission equipment (transmission lines, transformers, breakers, etc.) with correlated failure rates. For simplicity a transmission unit is represented with a 2-state Markov model. This model is very general and capable of representing single circuit outages, as well as common mode outages, station equipment failures, and protection system failures. As for units, state 1 is the UP state and state 2 is the DOWN state. The probability of state 1 for transmission unit i is p_i.

3. Analytic Simulation Method Description

The proposed method provides probabilistic characterizations of circuit flow and bus voltage magnitudes for the given electric load and generation system model. Specifically, the probability distribution function of circuit flow S_ℓ and the bus voltage magnitude V_i for each circuit ℓ and bus i is computed. This is achieved with a two step procedure. In the first step, the electric load and generating system model is used to characterize the power injections, Y, at the system buses as random variables. This is illustrated in Figure 2b. The random variables, Y, are in general correlated. Subsequently, a probabilistic power flow provides the probability distribution function of S_ℓ, V_i from the probabilistic model of the injection Y. Details are provided next.

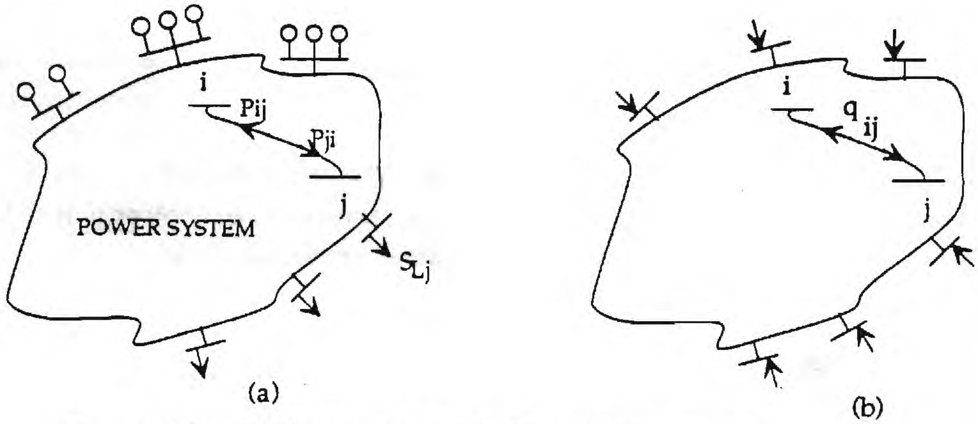


Figure 2. Schematic Representation of an Electric Power System
(a) Electric Power System Comprising Generation, Transmission and Loads
(b) Replacement of Generation and Loads with Injection Variables

3.1 <u>Generation System Simulation Method</u>

The objective of the generation system simulation method is to provide a probabilistic description of the bus power injections by taking into consideration the major operational practices and constraints of the system. Example of operating practices are: (a) economic dispatch, (b) dispatch of hydro units, (c) interchange schedules, (d) remedial actions, (e) security dispatch, (f) other. Examples of constraints are: (a) unit forced outages, (b) unit maintenance, (c) available hydro energy, (d) allowable range of bus voltages, (e) allowable loading level of circuits, etc. In subsequent paragraphs, the proposed simulation method will be outlined and an explanation will be given of how it accounts for operating practices and constraints.

Consider the proposed stochastic load model and the Markov model of generating unit availability. Further, consider the operation of the system during a specified period of time. This period of time may be contiguous or noncontiguous (for example, 1 pm to 3 pm each day for a period of one year). Utilizing the proposed electric load model, the electric load of a bus, during the period of simulation, can be characterized as a random variable with a probability distribution function and correlation to other bus loads. The total electric load can be characterized as a random variable, L, with a probability distribution function $F_L(\ell)$. Similarly, utilizing the Markov model of generating units, the unit availability can be represented as a discrete random variable. Assume that the n units of the system operate at level $x_1, x_2, ..., x_n$, respectively, while the total electric load is ℓ. If unit k is not in operation, then obviously x_k equals 0. Since there is a finite probability that any unit can be forced out, the output of unit j, x_j, can be considered to be a random variable with probability of force outage equal to q_j. We write

$$\Pr(A_j = x_j) = 1 - q_j,\ x_j \neq 0 \tag{9}$$
$$\Pr(A_j = 0) = q_j \tag{10}$$

Where A_j is a random variable representing the available capacity of unit j, q_j is the probability of unavailability. The above relationships state that the probability that the output of generator j is x_j equals $1 - q_j$, and the probability that the same output is zero equals q_j.

For the condition that has been considered, the apparent load ℓ_a will be

$$\ell_a = \ell - x_1 - x_2 - \dots - x_n \tag{11}$$

Since ℓ, x_1, x_2, ... x_n, are not deterministically known, the above equation can be replaced with its equivalent equation in term of the corresponding random variables

$$L_a = L - A_1 - A_2 - \dots - A_n \tag{12}$$

Where L is a random variable representing the local electric load and A_i is a random variable representing the output of unit i. Since the probability distributions of the random variable L, A_1, A_2, ..., A_n are known and since these random variables are independent, the probability distribution function of the random variable L_a is easily computed with a series of convolutions.

If we assume that $\ell_a > 0$ (that is, load exceeds generation), then another unit should be brought into operation or one or more of the operating units should increase their output. Assume that the unit i is operating at x_i and that it is selected according to a criterion to respond to any increases in the load. We shall refer to this criterion as the dispatch criterion. It is defined as to satisfy operational practices and constraints. The dispatch criterion will be qualified later. Without loss of generality, x_i may be equal to zero. In general, if $\ell_a > 0$, the output of unit i will increase from x_i to $x_i + \Delta x_i$, where Δx_i is a small increment (1-2 MW). We shall refer to this increment as the block Δx_i. The described formulation and direct application of probability theory, yields expressions for the expected energy, cost of operation, required fuel, etc., from the Δx_i increase in the output of generator i. The detailed mathematical formulation is given in [1].

Upon completion of the simulation algorithm, the probability that unit i operates at level x_i or the probability density function of power injections at the generation buses as well as the joint probability density of any generating unit pair has been constructed. Thus, the power injections to the electric power network are characterized as random variables with known probability distribution functions and correlations. It should be emphasized that the probability distribution function at generation buses can not be approximated with Guassian distributions. This basic result can also provide the performance parameters for each generating unit, such as: (a) expected time of operation, (b) expected produced energy, (c) expected production cost, etc. [1].

The proposed method is also capable of simulating the operating practices and constrains of a power system. This objective is achieved by appropriate selection of the dispatch criterion mentioned in the description of the method. Specifically, the dispatch criterion is defined as the sum of the actual operating cost of the generating unit plus a nonlinear penalty function of generating unit output defined with m parameters. In this way any arbitrary dispatch criterion can be accomodated. the parameters of the penalty are selected with a probabilistic optimization method which is described later. The solution of the optimization problem provides the parameters of the penalty function, such as the operational practices and constraints will be satisfied with maximum probability. Note that if the nonlinear penalty function is neglected, then the dispatch criterion equals the actual operating cost of the generating unit. This selection amounts to simulating the economic dispatch only, neglecting other operating practices, such as interchange schedules, etc., and operating constraints, such as hydro energy limitations, transmission limitations, etc.

3.2 Transmission System Simulation Method

The simulation method of a transmission system consists of transforming the probability density function of bus power injections into the probability density function of system output variables such as circuit loading, bus voltages, and system losses. This transformation can be achieved with any desirable degree of accuracy. Subsequently, the probability density functions of system output variables can be easily translated into reliability indices, total system losses, etc.

For the purpose of computing the probability distribution functions of output variables, an extension of the method reported in [3] will be employed. Specifically, consider an output variable v_{oi}. The output variable v_{oi} may be a bus voltage, the power flow on a circuit, the total system losses, the net interchange, etc. This variable is expressed as a function of the power injections:

$$v_{oi} = \bar{v}_{oi} + \sum_j f_{oij}(x_j - \bar{x}_j) \qquad (13)$$

where

$\bar{x}_{ji}$ is the expected value of the power injection at bus j (x may represent real or reactive power)

x_j is the power injection at bus j

$f_{oij}(\cdot)$ is, in general, a nonlinear function of the argument.

For practical calculations, the function $f_{oij}(\cdot)$ is expressed as a piecewise linear function:

$$f_{oij}(x_j - \bar{x}_j) = \{ s^k_{oij}\, y;\ x_j^{k-1} - \bar{x}_j \le y \le x_j^k - \bar{x}_j;\ k=1,2,... \} \qquad (14)$$

where

$$s^k_{oij} = dv_{oi} / dx_j \qquad \text{computed at } y = (x_j^{k-1} + x_j^k - 2\bar{x}_j)/2$$

References [3] and [6] provide an efficient procedure for computing the sensitivities s^k_{oij} based on the costate (adjoint) equation.

Equation (14) states that the function $f_{oij}(\cdot)$ can be considered linear in an interval $[x_j^{k-1} - \bar{x}_j, x_j^k - \bar{x}_j]$. The interval of validity of the linearized model depends on the parameters of the system in the neighborhood of bus j. A simple method for selecting the bounds $(x_j^{k-1} - \bar{x}_j)$ and $(x_j^k - \bar{x}_j)$ has been developed and reported in [6]. The method is based on the observation that the error is approximately an invariant function of the quantities $(P_{inj}/\Sigma Y)$ and $(Q_{inj}/\Sigma Y)$. where P_{inj} and Q_{inj} are real and reactive power injections at a bus and ΣY is the sum of admittances connected to the bus. For a given allowable error, these quantities must not exceed the threshold values ε_1 and ε_2 yielding the bounds $(x_j^{k-1} - \bar{x}_j)$ and $(x_j^k - \bar{x}_j)$. It is important to note that the interval of validity of the linearized model is quite wide even for small allowable threshold values, i.e., 2%. Practically, this means that variations of bus electric loads can be accurately represented with only one segment of the piecewise linear equation (14). More than one segment is needed only at generation buses, or buses with very large load, where the power injection variations are high.

Within the simulation method, the effects of circuit outages can be accounted for. For this purpose, the nonlinear effects of circuit outages are be accounted with a two step procedure: In the first step, the most important circuit outages are be identified with a ranking algorithm

[4]. In the second step, a probabilistic load flow computation is performed for a set of highly ranked circuit outages.

4. Method Validation

4.1 Example Test System

A three bus system has been used as the example test system to validate the proposed analytic simulation method for the composite power system. The system network is illustrated in the Figure 3. The system data is described following.

a) Electric Load Model

The electric load model of the test system is

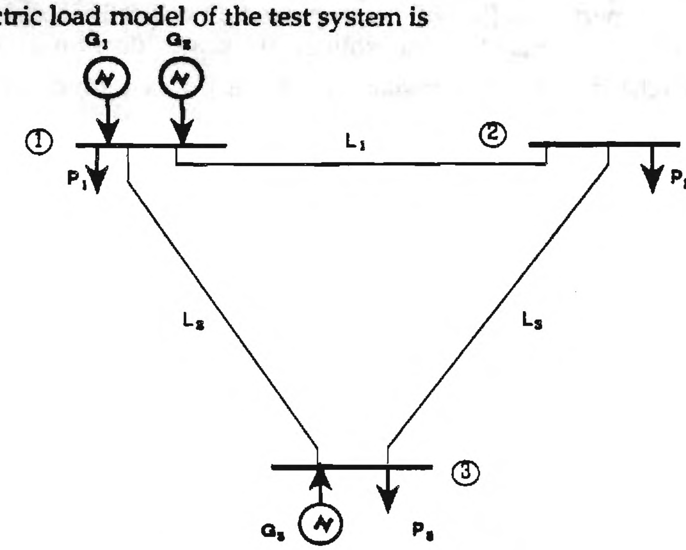


Figure 3. Example Three Bus System

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} 50 \\ 80 \\ 120 \end{pmatrix} + \begin{pmatrix} 25 & 20 \\ 15 & 18 \\ 20 & 5 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Where v_1, v_2 are independent Guassian variables with

$E[v_i] = 0$

$Var(v_i) = 1.0$

b) The Generating System model:

Unit No.	Pg,max(MW)	Cost Coefficient a	b	c	Availability
1	200	0	20	0.02	0.90
2	300	0	0	0	1.0
3	150	0	22	0.02	0.95

c) The Transmission System Model:

For purposes of validating the proposed method, we use a simplified DC network model. In this model the real power flow through the circuit can be represented as:

$$P_{lm} = \gamma_{lm} (\delta_l - \delta_m)$$

where

P_{lm} is real power flow from terminal l to terminal m

δ_l is the voltage phase angle at terminal l

δ_m is the voltage phase angle at terminal m

γ_{lm} is the transmission unit constant which is approximately equal to the total admittance of the transmission unit

The example test system has the following transmission system model.

Line L_1 :	$p_1 = 1.0$	$\gamma_1 = 15$
Line L_2 :	$p_2 = 1.0$	$\gamma_2 = 12$
Line L_3 :	$p_3 = 0.97$	$\gamma_3 = 8$

4.2 Sample Results

With the given electric load model, the total electric load is represented with

$$L = 250 + 60v_1 + 43v_2$$

Then the generating unit outputs are represented as a linear combination of the independent random variables v1 and v2 by taking into consideration the economic dispatch effect. Subsequently, generation bus power injections are easily expressed from the generating unit outputs. For the simplified three bus system, the result can be expressed with analytical functions. For example the output of unit 1 can be expressed as an analytical function of random variable v_1 and v_2 with respect to the economic dispatch.

$$P_{G1} = \begin{cases} 0 & 60v_1 + 43v_2 \le 50 \\ 60v_1 + 43v_2 - 50 & 50 < 60v_1 + 43v_2 \le 100 \\ 30v_1 + 21.5v_2 & 60v_1 + 43v_2 > 100 \end{cases}$$

Since v_1 and v_2 are Gaussain distributed, the distribution function of unit 1 can be represented by the standard Gaussian distribution.

$$F_{G1}(x) = \Pr\{P_{G1} \le x\} = \begin{cases} 0 & x < 0 \\ \Phi\left(\dfrac{x+50}{73.82}\right) & 0 \le x < 50 \\ \Phi\left(\dfrac{x}{36.91}\right) & x \ge 50 \end{cases}$$

where: $\Phi(\cdot)$ is the standard Guassian distribution.

Similarly, probability distribution functions can be exactly computed for other quantities. On the other hand the same probability distribution functions can be computed with the proposed analytic method and subsequently compared to the exact result. Examples of this comparison follow. Figure 4 shows the probability distribution of the power injection at bus 1 computed with the proposed method. The computed distribution using exact equations for this simple example are superimposed with dashed lines. Figure 5 illustrates the circuit power flow distribution. Again the results of the simulation method are superimposed on the exact results. It is obviously the proposed method provides has the good agreement with exactly computed results.

5. Conclusions

An analytic method for composite power system simulation has been described. A simple example was utilized to validate the method. For the simple example, the exact probability distribution functions have been computed and compared to results of the method. The comparison is favorable.

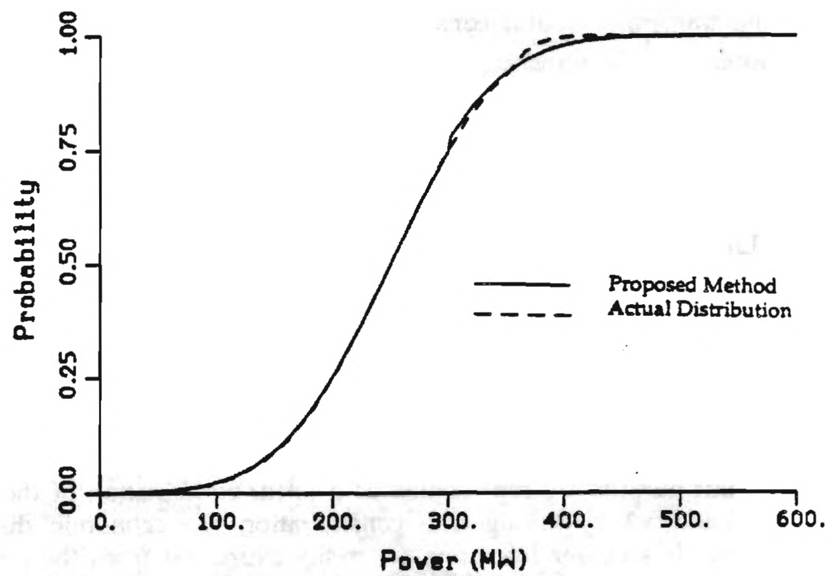


Figure 4. Probability Distribution Function of the Power injection at Bus1

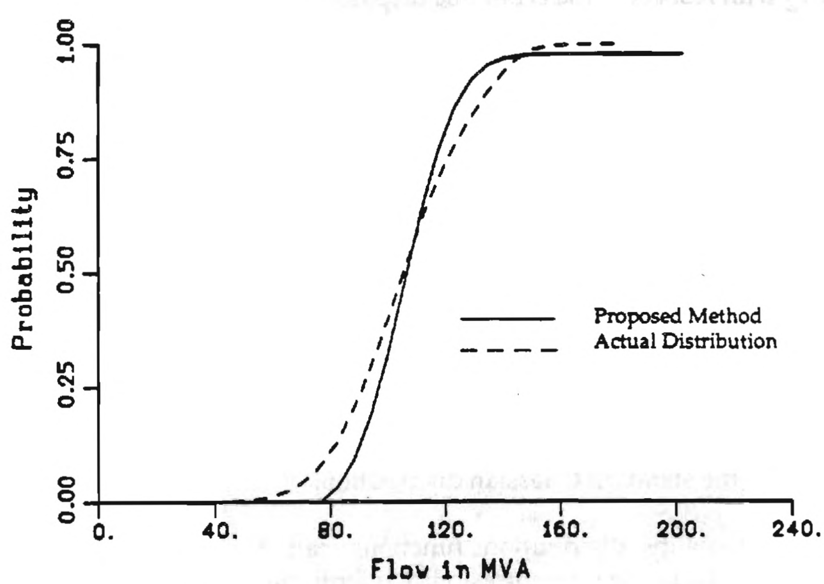


Figure 5. Probability Distribution Function of the Circuit 1 - 3 Power Flow

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